The Nature of the Fifth Dimension in Classical Relativity

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Abstract. Hyperdimensional unified field theories date back to 1921, when Kaluza reported that the Einstein equations of general relativity written in five dimensions (5D) would yield both the standard four-dimensional Einstein equations as well as the Maxwell equations, thus unifying general relativity and electrodynamics. Subsequent research focused on the field equations, with little attention paid to the equations of motion. While unified field theories have since evolved to address quantum forces with many extra compactified dimensions, the original classical 5D theory of Kaluza makes some compelling predictions about a macroscopic fifth dimension. These include identification of electric charge with the space/time manifestation of motion in the fifth dimension, emergence of the gravitational constant as a cosmic charge-to-mass ratio, and the prediction of hyperluminal 5D proper velocities. This paper provides the mathematical derivation of these and other results with a focus on the equations of motion, a critical consideration of the underpinnings and assumptions of the 5D theory, and a discussion of the historical context of the development of the 5D Kaluza theory and how its serious consideration could have been neglected for so long.

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INTRODUCTION

The search for a breakthrough in propulsion physics can be constrained by two key aspects to be expected in the new theory. One is that the theory must address the limits on interstellar travel implied by the classical theory of relativity. Conventional acceleration by any source of propulsion would still express time dilation effects and subluminal speeds.

They other key aspect is that the means of alternative transportation must be achievable through technologies which are ultimately based in our mastery of the classical electromagnetic field. We can never expect to build machines which are gravitationally dominant, nor can we expect to build machines by wielding the strong and weak quantum forces. Our manufacturing, chemistry, and communications technologies are based on the electromagnetic field. A sensible theory of breakthrough propulsion may therefore be expected to somehow extend both general relativity and electrodynamics.

THE PROSPECTS OF CLASSICAL 5D RELATIVITY

Soon after Einstein published the general theory of relativity (GR), Kaluza (1921) discovered that casting Einstein's theory in five dimensions (5D) instead of the usual four would yield both four-dimensional (4D) GR and the 4D Maxwell equations of electrodynamics.

The fundamental entity in 5D GR is the symmetric 5D metric tensor, which has 5 more components than the 4D metric tensor. Kaluza theory identifies the 4 new off-diagonal components with the electromagnetic potential 4-vector. The fifth component is a scalar field with no compelling tie to 4D physics. It can be interpreted within a framework similar to that developed by Brans and Dicke (1961), in which GR is augmented with a scalar field

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identified as the gravitational constant *G*. Kaluza (1921) originally made the simplifying assumption that the scalar field had no spacetime variation.

The 5D theory would seem to offer an avenue toward breakthrough physics because it satisfies the expectations for such a theory as well as hinting at a possible approach to hyperluminal travel: the fifth dimension. The fifth dimension offers the prospect that different points in space can be traversed in a dimension other than time, thereby defeating the limitations of 4D travel.

The 5D Kaluza theory began to fall out of favor with the advent of the quantum revolution, when it was expected a fundamental theory would be a quantum theory. Kaluza theory began to follow a quantum trajectory early on when Klein (1926a) proposed a quantum interpretation of Kaluza's 5D theory. Since then, the idea of multiple dimensions has been used to construct unified quantum field theories. Of course, these theories require many extra dimensions, and they are always assumed to be compactified and microscopic. Although quantum electrodynamics has been a great success, quantum relativity has still eluded us and one begins to wonder if a quantum theory of GR is possible at all.

Historically, the focus on Kaluza theory has been the field equations. Yet there exists another leg of physical law which bears on breakthrough propulsion: the equations of motion. The equations of motion in GR are given by the geodesic hypothesis and are independent of the Einstein equations for the field. The 5D equations of motion produce the Lorentz force for particles in electromagnetic fields. It has been little-appreciated but motion in the fifth dimension is identified with electric charge.

A historical issue facing the Kaluza theory was that the field equations were discovered a quarter century after Kaluza's original paper and the early classical work. Their discovery seemed to falsify Kaluza's assumption of a constant scalar field, since the field equations predicted electromagnetic fields were source terms in the scalar field equation. Assuming a constant scalar field, although observationally motivated, would then be tantamount to assuming constant electromagnetic fields. In this paper it is shown that the scalar field can only vary on cosmological scales, implying that a constant scalar field is indeed a valid approximation to non-cosmological applications of the Kaluza theory.

The classical 5D theory has not been falsified and seems to offer a promising approach to breakthrough propulsion. The theory is outlined and its relation to 4D theory discussed. The equations of motion and the nature of the fifth dimension in particular are considered in this paper.

THE CYLINDER CONDITION

Kaluza relativity provides many more degrees of freedom than are necessary to reconstitute 4D GR and electrodynamics. The simplest form of the 5D theory is to assume that the 5D metric tensor does not depend on the fifth coordinate, an assumption historically known as the cylinder condition. Kaluza (1921) used it in his original paper. The cylinder condition originally met with some skepticism when the 5D theory was proposed, as it seemed somehow ad hoc. In an age of dark energy and cosmological constants, it does not seem so offensive.

The cylinder condition can be understood as a boundary condition reflective of the fact that we do not perceive or measure a fifth dimension. Yet the fifth dimension is still understood to be macroscopic. And all of 4D GR and electrodynamics can be recovered under the cylinder condition. It may be that breakthrough propulsion in the 5D theory, if it could be achieved at all, would come from a dependence of the fields on the fifth coordinate. Yet it is anticipated that the nature of the fifth dimension can be illustrated without recourse to how fields may vary with the fifth coordinate, so the observationally-motivated mathematical expedient of the cylinder condition is assumed in this paper.

5D LINE ELEMENT

The following notation is adopted. Five-dimensional tensors are indicated with a tilde to distinguish them from four dimensional ones. The time coordinate is x^0, and the spatial coordinates x^1, x^2, and x^3. The fifth coordinate is x^5. Summation is implied on repeated pairs of covariant and contravariant indices. Roman indices range over all five coordinates, and greek indices over the usual four coordinates of space and time. Partial derivatives $\partial/\partial x^a$ are abbreviated ∂_a.

Classical Kaluza relativity comprises 3 sets of equations: the form of the 5D metric, the 5D Einstein equations for the metric, and the 5D geodesic hypothesis for the metric. In a space described by the 5D metric $\tilde{g}_{ab}$, particles of matter move along 5D paths parameterized by a 5D proper distance s:

$$\tilde{g}_{ab}dx^a dx^b \equiv ds^2 , \tag{1}$$

which constrains the 5D proper velocity, $\tilde{U}^a \equiv dx^a/ds$. The coordinates x^a have units of length as does the proper distance s, implying $\tilde{U}^a$ is dimensionless.

5D METRIC

The 4D metric $g_{\mu\nu}$ is related to the 4D proper time τ such that $g_{\mu\nu}dx^\mu dx^\nu = c^2 d\tau^2$, where c is the speed of light. The Kaluza ansatz is to posit the 5D metric $\tilde{g}_{ab}$ in terms of the 4D metric $g_{\mu\nu}$ and the electromagnetic vector potential A^μ :

$$\begin{aligned} \tilde{g}_{\mu\nu} &= g_{\mu\nu} + k^2\phi^2 A_\mu A_\nu \qquad \tilde{g}^{\mu\nu} = g^{\mu\nu} \\ \tilde{g}_{5\nu} &= k\phi^2 A_\nu \qquad \tilde{g}^{5\mu} = -kA^\mu \\ \tilde{g}_{55} &\equiv \phi^2 \qquad \tilde{g}^{55} = \frac{1}{\phi^2} + k^2 A^2 \end{aligned} \tag{2}$$

where $A^2 \equiv A_\alpha A^\alpha = g_{\alpha\beta}A^\alpha A^\beta$, $\tilde{g}_{ab}\tilde{g}^{bc} = \delta^c_a$, and k is a constant which will be seen to scale with the gravitational constant. The price of unifying $g_{\mu\nu}$ and A_μ within the framework of a 5D metric is the necessary introduction of the scalar field ϕ .

The 5D metric (2) was studied in a series of classic papers including Kaluza (1921), Klein (1926a), and Thiry (1948). Closely related papers include Klein (1926b), Jordan (1947), and Brans and Dicke (1961). Bergmann (1942), Bargmann (1957), Applequist *et al*. (1987), and Overduin and Wesson (1997) provide reviews.

The signature of the fifth dimension in the metric must be spacelike (Bergmann, 1942; Bargmann, 1957). The entire space x^a is hyperbolic, but the subspace of (x^1, x^2, x^3, x^5) is Euclidean.

Note that equations (1) and (2) imply a relation between the 4D and 5D line elements:

$$ds^2 = c^2 d\tau^2 + \phi^2 \left(kA_\nu dx^\nu + dx^5\right)^2 . \tag{3}$$

5D FIELD EQUATIONS

The simplest form of Kaluza theory, and the one originally put forth by Kaluza (1921), assumes $\partial\tilde{g}_{ab}/\partial x^5 = 0$, the relation historically known as the cylinder condition. This condition is assumed *a priori* here merely as the minimal version of Kaluza theory necessary to recover standard 4D physics. Overduin and Wesson (1997) consider the implications of relaxing the cylinder condition.

The 5D field equations are obtained from the 5D Einstein equations, $\tilde{G}_{ab} \equiv \tilde{R}_{ab} - \tilde{g}_{ab}\tilde{R}/2 = 0$, where $\tilde{R}_{ab}$ is the 5D Ricci tensor, and $\tilde{R}$ is the 5D scalar curvature. Kaluza originally assumed ϕ to be constant and it was not until some twenty five years later that Thiry (1948) obtained the full 5D field equations:

$$\tilde{G}_{\mu\nu} = G_{\mu\nu} - \frac{k^2\phi^2}{2}T^{EM}_{\mu\nu} - T^{\phi}_{\mu\nu} = 0 \ , \tag{4}$$

$$\nabla^{\mu}F_{\mu\nu} = -3g^{\mu\alpha}F_{\mu\nu}\partial_{\alpha}\ln\phi \ , \tag{5}$$

$$\phi^{-1}g^{\alpha\beta}\nabla_{\alpha}\nabla_{\beta}\phi = \frac{k^2\phi^2}{4}F_{\alpha\beta}F^{\alpha\beta} \ , \tag{6}$$

where $\nabla\alpha$ is the 4D covariant derivative, $G\mu\nu$ is the standard 4D Einstein tensor, $F_{\mu\nu} \equiv \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$, $T^{TEM}_{\mu\nu}$ is the standard electromagnetic stress tensor, and $T^{\phi}_{\mu\nu} \equiv \phi^{-1}\left[\nabla_{\mu}\nabla_{\nu}\phi - g_{\mu\nu}g^{\alpha\beta}\nabla_{\alpha}\nabla_{\beta}\phi\right]$.

In Kaluza theory the constant k is fixed by correspondence of the 5D vacuum Einstein equations (4) to the 4D Einstein equations are

$$G_{\mu\nu} = \frac{8\pi G}{c^4}T^{EM}_{\mu\nu} \ , \tag{7}$$

so equations (4) and (7) imply

$$\phi^2k^2 \equiv \frac{16\pi G}{c^4} \ . \tag{8}$$

The 4D Einstein equations (4) with the extra source term arising from ϕ, along with the field equation (6) for ϕ, are quite similar to the standard Brans-Dicke theory which augments general relativity with a scalar field. Unique to Kaluza theory is the field equation (6) for ϕ and its surprising dependence on the electromagnetic field. As suggested by Brans and Dicke (1961) and Jordan (1947), and implied by equation (8), ϕ may be interpreted in terms of the gravitational constant G. Specifically,

$$k^2 \equiv \frac{16\pi G_0}{c^4} \qquad \phi^2 = G/G_0 \ , \tag{9}$$

where G_0 is a normalizing constant.

Fitting the scalar field into a coherent framework of understanding that still provides the 4D classical limits has long been a challenge to Kaluza relativity. In fact, as summarized by Applequist *et al.* (1987), the significance of ϕ in the field equations was not fully appreciated until some two decades after Kaluza's paper. Although Kaluza originally assumed ϕ constant, it was not realized until Thiry (1948) obtained the full field equations the implied constraint of equation (6) on the electromagnetic field from assuming constant ϕ.

Upon closer inspection, the magnitude of k given by equation (9) implies that setting equation (6) to zero is not a meaningful constraint on terrestrial electromagnetic fields and that constant ϕ is a valid assumption on non-cosmological scales.

To see this, note that the source term for ϕ on the right hand side of equation (6) is a Lorentz scalar representing the difference of the energy densities in the electric and magnetic fields. For electromagnetic fields in which energy is partitioned equally among electric and magnetic components, this term will vanish. But consider situations of strong astrophysical electromagnetic fields which may maximize this term. The field equation (6) for ϕ and the equation (8) for k imply a scale of variation for ϕ given by $l_{\phi} \sim \left(k^2\phi^2\varepsilon_{EM}\right)^{-1/2} \sim c^2/7\left(G\varepsilon_{EM}\right)^{1/2}$, where ε_{EM} is electromagnetic energy density. The scale of variation of ϕ arising from a neutron-star magnetic field of 10^{12} Gauss would be of order 1 astronomical unit. To achieve variations in ϕ on a scale of kilometers would require energy densities of order 10^{38} erg cm^{-3} or magnetic field strengths of order 10^{20} Gauss. For an electromagnetic energy density similar to the cosmic microwave background value of 0.25 eV cm^{-3}, the magnitude of k implies ϕ varies on a length-scale similar to the radius of the universe.

In summary, ϕ can be approximated as a constant in the equations of motion of terrestrial objects, and the constraint of equation (6) from assuming constant ϕ is understood to be only on the cosmological electromagnetic energy density. Kaluza's original assumption of constant ϕ is indeed valid in the equations of motion of terrestrial objects and provides no constraint on non-cosmological electromagnetic fields.

COSMOLOGICAL IMPLICATIONS

If ϕ, and therefore G, is truly a cosmological field, let us consider the field equations (4) and (6) when $g_{\mu\nu}$ is the standard Robertson-Walker metric, describing a homogeneous and isotropic universe. Further limit consideration to that of a flat universe, consistent with current observations. The non-zero components of $g_{\mu\nu}$ are:

$$g_{tt} = c^2 \quad g_{rr} = -a^2 \quad g_{\theta\theta} = -a^2 r^2 \quad g_{\psi\psi} = -a^2 r^2 \sin^2\theta$$

where r, θ, and ψ are comoving spherical coordinates, t is the cosmological proper time, and $a(t)$ is the expansion scale factor. Demand ϕ is homogeneous and isotropic so that $\phi = \phi(t)$ only.

Then the G_{tt} component of equation (4) reproduces the Friedmann equation for a flat universe, but with a varying gravitational constant:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_0}{3c^2}\phi^2 \varepsilon_{EM} \tag{10}$$

where $T_{tt}^{EM} \equiv \varepsilon_{EM} \cdot T_{tt}^{\phi} = 0$ and makes no contribution to the Friedmann equation.

The corresponding equation for $\phi(t)$ is provided by equation (6):

$$\frac{\ddot{\phi}}{\phi^3} = \frac{4\pi G_0}{c^2} F^2 \tag{11}$$

where $F^2 \equiv F_{\alpha\phi} F^{\alpha\beta}$ is also a function of t.

Since we have not considered matter, these equations only apply to the early radiation-dominated universe. The dominant electromagnetic factor in these equations arises from the cosmic microwave background (CMB), and its energy density $\varepsilon_{CMB} \propto a^{-4}$. For an isotropic radiation field like the CMB, energy is expected to be partitioned equally among magnetic and electric field oscillations so that $F_{CMB}^2 = 0$. In this case, $\phi \propto t$ and the gravitational constant $G(t) \propto t^2$, although constant-value solutions to equation (11) are also allowed. The modified Friedmann equation (10) then implies $a(t) \propto t$, whereas the standard Friedmann result for a radiation dominated universe is $a(t) \propto t^{1/2}$.

So equation (6) therefore implies the gravitational constant G is a function of the cosmological electromagnetic field, and G_0 refers to its value in the present epoch of cosmological electromagnetic energy density. The scale of variation $l_\phi \propto t^{-1}$.

In fact, findings such as CMB anisotropy data (Nagata, Chiba and Sugiyama, 2004) and nucleosynthesis data (Copi, Davis, and Krauss, 2004) imply that G could not have varied by more than a few percent or so since the Big Bang. Therefore the $G(t) \propto t^2$ solution of equation (11) must be rejected and the constant solution taken in order to correspond to observation, recovering the standard result for a radiation-dominated universe.

5D EQUATIONS OF MOTION

The Kaluza metric (2) provides the Lorentz force from the 5D equations of motion as well as the Maxwell equations from the 5D field equations. It is a marvel of Kaluza relativity that the same metric has the right behavior in the two independent sets of equations.

Independent of the field equations, the 5D equations of motion obtain from the 5D geodesic hypothesis:

$$\tilde{U}^b \nabla_b \tilde{U}^a \equiv \frac{d\tilde{U}^a}{ds} + \tilde{\Gamma}^a_{bc} \tilde{U}^b \tilde{U}^c = 0 \tag{12}$$

where $\tilde{\nabla}_a$ is the 5D covariant derivative. Γ^a_{bc} is the 5D affine connection, which has the standard relation to the metric:

$$2\Gamma^a_{bc} = \tilde{g}^{ad}\left[\partial_c \tilde{g}_{bd} + \partial_b \tilde{g}_{cd} - \partial_d \tilde{g}_{bc}\right] \tag{13}$$

The modified 4D equations of motion are given by the 4D spacetime components of equation (12), rewritten in terms of the 4D proper velocity $U^\nu \equiv dx^\nu / d\tau$:

$$\frac{dU^\nu}{d\tau} + \tilde{\Gamma}^\nu_{55}\left(U^5\right)^2 + 2\tilde{\Gamma}^\nu_{5\mu} U^5 U^\mu + \tilde{\Gamma}^\nu_{\alpha\beta} U^\alpha U^\beta + U^\nu \frac{d}{d\tau}\ln\left(\frac{d\tau}{ds}\right) = 0 \tag{14}$$

Equation (14) is the generalized spacetime equation of motion. The first, third, and fourth terms obviously have the appropriate form to reproduce 4D physics. Before considering the terms of equation (14), consider the equation for $\tilde{U}^5$.

A standard result of GR provides the equations of motion (12) in an alternate form, for the covariant 5-velocity $\tilde{U}_a = \tilde{g}_{ab}\tilde{U}_b$:

$$\frac{d\tilde{U}_a}{ds} = \frac{1}{2}\tilde{U}^b \tilde{U}^c \frac{\partial \tilde{g}_{bc}}{\partial x^a} \tag{15}$$

This form of the equations of motion is in terms of a normal partial derivative instead of a covariant derivative, and expresses the conservation properties of the metric. There is a conserved quantity associated with each invariant coordinate.

A simple equation for $\tilde{U}^5$ is obtained by applying the cylinder condition $\partial \tilde{g}_{ab} / \partial x^5 = 0$ to equation (15):

$$\tilde{U}_5 = \tilde{g}_{5a}\tilde{U}^a = \phi k A_\nu \tilde{U}^\nu + \phi \tilde{U}^5 = \text{constant} \tag{16}$$

along 5D world lines. Note that equations (3) and (16) imply $d\tau/ds$ is constant.

THE NATURE OF ELECTRIC CHARGE

Now consider correspondence of equation (14) to the Lorentz force. In the 4D equations of motion (14), the term linear in U^μ must correspond to the Lorentz force. Under the twin assumptions of constant (cosmological) ϕ and the cylinder condition, we indeed find from equations (2), (9), and (13) that

$$2\tilde{\Gamma}^\nu_{5\mu} = k g^{\nu\alpha} F_{\mu\alpha} \tag{17}$$

In order to recover the standard expression for the Lorentz force, electric charge q (in cgs units) must be identified with U^5 such that

$$kU^5 = \frac{q}{mc} \tag{18}$$

where m is rest mass of the object in 4D motion and k is given by (9).

Since $d\tau/ds$ is constant, equations (16) and (18) can be rewritten in conventional 4D terms:

$$\frac{16\pi \mathrm{G}}{\mathrm{c}^3} m A_\nu U^\nu + q = \text{constant} \tag{19}$$

Electric charge is a frame-dependent quantity, and its variation with motion in electromagnetic fields is given by equation (19). For a relativistic proton with proper velocity ~100c, moving in a 10^6 Gauss magnetic field characterized by a length-scale of 10 meters, the variation in charge is ~ 10^{-38}. Therefore, while Kaluza relativity predicts electric charge is not a true Lorentz scalar; its variation with 4D motion is practically indistinguishable from a Lorentz scalar.

THE ENERGY-MOMENTUM-CHARGE 5-VECTOR

The conventional energy-momentum 4-vector is augmented in the 5D theory with a fifth component, electric charge. The electric charge of an object at rest in the laboratory arises from its motion in the fifth dimension. Just as energy arises from "motion" in time and momentum arises from motion in space, Kaluza relativity tells us that electric charge arises from "motion" along the 5th coordinate. Just as particle energy $\propto U_0$ and spatial momentum $\propto \boldsymbol{U}$ are frame-dependent, so is electric charge $\propto U^5$.

The equation for the fifth component of the 5-velocity under the cylinder condition is given by equation (19), which implies the existence of a rest charge. The rest charge values for elementary particles are essentially their tabulated values. When the electromagnetic field vanishes; electric charge is a true Lorentz scalar. However, a charged particle in 4D motion in an electromagnetic field will show a frame-dependence of electric charge as given by equation (19).

The gravitational constant takes on new significance in light of equations (9) and (18), which imply:

$$U^5 \sim c\frac{q/m}{G^{1/2}} \tag{20}$$

The Gaussian units of electric charge are $m^{1/2}l^{3/2}t^{-1}$, where m denotes a mass unit, l a length unit, and t a time unit. The units of G are $l^3m^{-1}t^{-2}$, so the gravitational constant provides a universal charge-to-mass ratio.

The charge component of the 5-vector can dominate the other components, just as the time component is typically larger than the spatial components of 4-vectors. For elementary particles U^5 can be hyperluminal: $10^{18}c$ for protons, and $10^{21}c$ for electrons. The gravitational constant sets a length-scale in the fifth dimension just as the speed of light sets a length-scale in the time dimension.

This creates an apparent problem in the equations of motion (14), one which was recognized by Kaluza. It is that the magnitude of U^5 for elementary charged particles would cause the term quadratic in U^5 in equation (14) to dominate the Lorentz and Einstein force terms linear and quadratic, respectively, in U^μ.

However, under the assumptions of the cylinder condition and of constant ϕ, the term in equation (14) quadratic in U^5 vanishes because $\tilde{\Gamma}^\nu_{55} = 0$.

PRESERVATION OF THE EQUIVALENCE PRINCIPLE

Brans and Dicke constructed their scalar-tensor theory of gravity to preserve the principle of the equivalence of gravitation and inertia. For Brans and Dicke, preservation of the equivalence principle meant not having the scalar field enter the equations of motion. Of course, the 5D equations of motion (12) manifestly obey a 5D equivalence principle. But in order to connect back to 4D theory we must consider the apparent violations of the equivalence principle in the 4D equations of motion (14).

It has already been established in the previous section that under the cylinder condition and assumption of constant ϕ, the second term in equation (14) vanishes. Under those same conditions, the third term gives the standard Lorentz force. And since $ds/d\tau =$ constant, the fifth term in equation (14) vanishes as well. Consider then the remaining term, the fourth.

For the cylinder condition and constant ϕ, the coefficient of the term quadratic in U^μ is given by:

$$\tilde{\Gamma}^\mu_{\alpha\beta} = \Gamma^\mu_{\alpha\beta} + \frac{k^2}{2}g^{\mu\nu}\left(A_\alpha F_{\beta\nu} + A_\beta F_{\alpha\nu}\right) \tag{21}$$

The first term on the right hand side of equation (21) is just the standard 4D affine connection. The second term in A_μ is the contribution of electromagnetic stresses to spacetime curvature and is of order k^2A^2. This effect is found in 4D GR since an electromagnetic stress tensor can be a source of spacetime curvature in the Einstein equations (7).

Note that the units of vector potential A^{μ} are $m^{1/2}l^{1/2}t^{-1}$. Therefore $k^2A^2 \sim GA^2/c^4$ is the dimensionless number that expresses the strength of the coupling corrections to the 4D theory. The factor k^2A^2 is a very small number, and quantifies weak couplings to electromagnetic fields. For a neutron star magnetic field of strength 10^{12} Gauss and length-scale 10 km, k^2A^2 is of order 10^{-13}. For galactic electromagnetic field energy densities of 1 eV/cm^3 and length scale 1 parsec, k^2A^2 is of order 10^{-32}.

Therefore, under the cylinder condition and constant scalar field assumptions, the 4D equations of motion are recovered in the 5D theory and the equivalence principle is preserved.

CONCLUSIONS

Because a breakthrough propulsion theory must assimilate classical relativity and electrodynamics, the 5D general relativity introduced by Kaluza remains a compelling prospect. This theory unifies 4D relativity and electrodynamics within a 5D metric. The presence of a fifth macroscopic dimension itself has compelling implications for breakthrough propulsion.

Assuming the form of the 5D metric allows recovery of the 4D Einstein and Maxwell equations from the 5D Einstein equations, as well as the 4D geodesic equations and the Lorentz force from the 5D geodesic equations. Yet the 5D theory contains enormous extra degrees of freedom, which must be constrained for a practical theory. The most basic of these simplifications has been called the cylinder condition, and is to assume the 5D metric does not depend on the fifth coordinate. Another is to assume the new scalar field, which is necessary in a 5D metric, is constant. Even under these most basic constraints, the 5D theory still reproduces the 4D theory as described above and makes some compelling predictions about the nature of the fifth dimension.

Five dimensional relativity identifies electric charge as the 4D manifestation of 'motion' in the fifth dimension. The proper velocities of elementary charged particles are hyperluminal, of order $10^{20}c$. Electric charge is seen to be the fifth component of an energy-momentum-charge 5-vector and therefore not a true Lorentz scalar.

Newton's gravitational constant enters the theory as a cosmological charge-to-mass ratio and quantifies the causal structure of the fifth dimension, just as the speed of light quantifies the relationship between space and time.

The scalar field of the 5D theory varies only on cosmological scales and so can justifiably be treated as constant in the field equations and equations of motion.

The 5D effects on the standard 4D equations of motion under these conditions are such that the equivalence principle is not violated. However, there is a new equation of motion for electric charge in which it may vary for motion of a charged body in an electromagnetic field.

Further research is required to evaluate whether motion in the fifth dimension could be engineered to provide a corresponding displacement in space but without the usual lapse in time accompanying 4D spatial displacements.

NOMENCLATURE

$\tilde{g}_{ab}$ = 5D metric tensor
$g_{\mu\nu}$ = 4D metric tensor
A^{μ} = electromagnetic vector potential
G = gravitational constant
q = electric charge
m = mass
s = 5D proper distance
τ = 4D proper time
c = speed of light

- greek indices α, β, μ, ν and so on run over the four space-time coordinate labels, 0, 1, 2, 3. The time dimension is coordinate 0 and the three spatial dimensions are coordinates 1, 2, 3.
- roman indices a, b, c, d and so on run over the 5 dimensional coordinate labels, 0, 1, 2, 3, 5, where coordinate 5 is the fifth dimension coordinate
- contravariant vectors have raised indices, such as U^a , while covariant vectors have lowered indices, such as U_a .
- summation is implied on repeated indices
- tildes ~ are used to indicate five-dimensional tensors

ACRONYMS

4D = four dimensional
5D = five dimensional
CMB = Cosmic Microwave Background
GR = General Relativity
eV = electron volt

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