Bringing Inertial Induction Theory in from the *Cold*

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Agenda
1. report from APS: how to go mainstream
2. parable of Sciama
3. technical: why there is no Maxwellian gravity

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Estes Park APW 11 Sept 2018

Why do you say "inertial induction"?

- "inertial induction" is the historical generic term referring to mechanical coupling with the distant universe
  - Used in the Brans 1977 paper on this topic
  - suggested as preferable to "Machian effects" and "Mach's principle", in the 1995 Barbour & Pfister proceedings on Mach's Principle
- The basic principle, if it exists, underlying propellantless propulsion advances within the framework of the accepted theory of gravity
  - *Local* gravitational exchange of momentum with the *distant* universe
- "Mach effect", MEGA, MAGA … are all meaningless terms to other scientists.
  - This is physics, not philosophy. We are talking about what we can measure

What do you mean by "the cold"?

- Working outside of the mainstream of science, and outside the consensus of physical law
- Avoiding presentations to skeptical peers (APS Division of Gravitational Physics)
- Avoiding publication in peer-reviewed journals of gravitational physics (Phys Rev, Nature, Science, ApJ)

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Why should I bother going to APS and publishing in Phys Rev?

- There are people at APS who have been studying gravity for 50 years – and this problem for 50 years!
- These are the people who found a way to detect a force of nature so minute that it moved an interference fringe 1/1000 the width of a proton!
- No one gets to the stars alone – unless your boat is your skull. These would be the first people to understand and validate the discovery of inertial induction in gravitational physics
- They can save you years of work and blind alleys

Avoiding these venues is scientific dereliction

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How to Bring Inertial Induction in from the Cold

- Inertial induction is a well-known topic among grav researchers, going back decades. We have only to pick up the thread
- But: According to Brans (1977), *Absence of Inertial Induction in General Relativity*, the Woodward device and concept is impossible for the reasons laid out in his paper, and this is the current mainstream view. The current accepted viewpoint as codified in Brans must be addressed and reconciled with peers.
- Our current theory of gravity, GR, is unassailable and triumphant. The path to a credible theory of inertial induction is narrow: it must depart from the known laws of gravity.
  - A theoretical derivation of inertial induction from the laws of gravity remains missing, and the principle unproven. The Woodward equation and derivation is inconclusive, because it does not depart from either the geodesic equation or the Einstein equations.
  - Alternative theories of gravitation like Hoyle-Narlikar will be considered premature because there is no experiment that is not explained by GR. An assumption of laws other than GR would be considered as arbitrary as assuming the solution itself.

- A search for inertial induction in linear GR must be abandoned (Sciama's dream).
  - Momentum exchange with weak gravitational fields occurs at second order in the perturbation, missing by definition from linear GR
  - A theory of inertial induction must be sought in an exact analytical solution to the Einstein equations, in a numerical simulation, or a perturbation treatment carried to second order.

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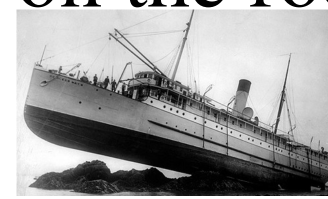
The Parable of Sciama

Perils of ignoring the known laws of gravity

- How a scientist became entranced by the sirens of Maxwell, and smashed his credibility on the rocks
  - Dennis Sciama



- How another scientist also became entranced by the Maxwell sirens, but dragged himself free by force of scientific scruple, and floated away on the Kerr metric
  - John Wheeler

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The Laws of Gravity
(Bow before them)

How gravity responds to matter
(field equations)

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi \frac{G}{c^4} T_{\mu\nu}$$

How matter responds to gravity
(force equation)

$$\nabla_\mu T^{\mu\nu} = 0$$

- The equations have passed numerous experimental tests
- No situation has gone unexplained by these equations
- However: the field equations are a *bitch*. 10 coupled, second-order, non-linear (quadratic) partial differential equations
  - Exact analytic solutions are as rare as hen’s teeth, with high symmetry only

Any theoretical investigation of inertial induction MUST depart from these equations

Milestones in GR

- 1915: Field equations of GR
  - Non-linear, second-order partial differential equations in 10 gravitational field variables
- 1916: Schwarzschild metric: exact solution for the gravity of a mass
- Before 1920: deflection of starlight, perihelion of Mercury, gravitational redshift, gravitational waves
- 1925-1955: desert of GR

- 1957: Chapel Hill Conference: the rains finally come
- 1960: Kruskal transformation of Schwarzschild metric
  - Understanding of the singularity at the Schwarzschild event horizon
- 1963: Kerr metric: exact solution for the gravity of a rotating mass
- 1965: Newman metric: exact solution for the gravity of a charged, rotating mass
- 1970-2000: development of numerical relativity software
  - Solution of the GR two-body problem: colliding black holes
  - Elliptic equations on the boundary, hyperbolic equations on the domain
- 1972-present: availability of good textbooks on GR, covering linear theory, gauge choices, etc
- 2016: detection of gravitational waves

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Out of the desert, comes a man with a vision

- Dennis Sciama, 1953, MNRAS:
  - *Wow. The gravity equations are really tough. I don't know how to solve them.*
  - *I know! I'll just assume gravity obeys the Maxwell equations. I can solve those!*
  - *Holy cow! If gravity obeys the Maxwell equations, I can explain inertia!*
  - *Therefore, gravity explains inertia!*
- 1957: *Oops, I did it again*: Scientific American article
  - A perfect venue for hand-waving: no pesky equations in Sci Am

Laws of Electromagnetism

How electromagnetic fields respond to matter – Maxwell Eqns

$$\partial_\mu F^{\mu\nu} = 4\pi \frac{J^\nu}{c}$$

How matter responds to electromagnetic fields – Lorentz Force

$$\frac{d^2 x^\mu}{d\tau^2} = \frac{q}{mc} F^{\mu\nu} \frac{dx_\nu}{d\tau}$$

- Field equations named for Maxwell, but containing the laws of Ampere, Gauss, Coulomb, and Faraday
- Unique features: electromagnetic radiation and electromagnetic induction

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But seriously, Dennis – gravity is nothing like electromagnetism

Gravity	Electromagnetism
Different masses fall the same	Different charges fall differently
Spacetime is dynamic	Spacetime is static
$\frac{d^2 x^\mu}{d\tau^2} \propto \frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}$	$\frac{d^2 x^\mu}{d\tau^2} \propto \frac{dx^\nu}{d\tau}$
Field equations non-linear	Field equations linear
quadropole	dipole
10 components of the gravitational potential	4 components of the electromagnetic potential
Grav fields feel grav, Like everything else	EM fields don't feel EM (EM fields aren't charged)

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The cheese slips from the cracker:

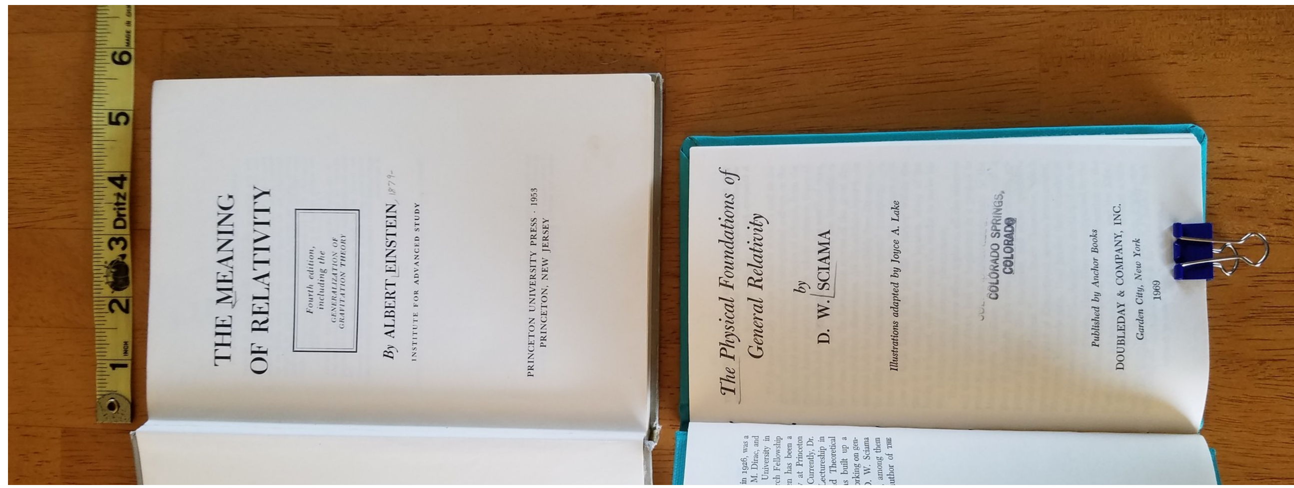
1969, The Physical Foundations of General Relativity

No Einstein equations,
only Newton's laws!

Devoted to the arguments of 1953

This time, without the indefensible
Maxwellian assumption

Now it's pure hand-waving



Yet Sciama admits at the end:

"Our previous calc'n of the total inertial force due to all the matter in the universe is neither strictly correct nor entirely correctable.

"We can only hope that our linear approx'n gives an answer with the correct order of magnitude."

The puzzle of Sciama

- Why didn't he just depart from the known laws of gravity?
- Why didn't he find a suitable Maxwellian limit in the jungle of solutions to GR?
- Did he forget his duty as a scientist to work from a consensus picture of reality?
- Epilogue:
  - 1989: *Sciama interview with Alan Lightman*, for the AIP oral history project, discussing his work on Mach's principle:
    - "I liked [Mach's principle], once I learned the idea. And I was very disappointed when it all went into the sand."
  - 1995: *Ciufolini & Wheeler:*
    - "An eminent physicist spent the last years of his life pushing the idea that gravitation follows the pattern of EM. This thesis we cannot accept, and the community of physics, quite rightly, does not accept"

Wheeler meets the sirens

- John A. Wheeler: luminary of 20th century physics
- Co-author of an early comprehensive textbook on GR (1973)
  - “matter tells space how to curve, space tells matter how to move”
- Co-authored *Gravitation & Inertia*, 1995
  - Compact mathematical introduction to GR
  - AND addresses problem of inertia in GR
    - “mass there governs inertia here”
    - Development of a complex mathematical “initial-value” framework
  - Final chapter addresses the Sciama result and its Maxwellian underpinnings

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Wheeler escapes the sirens: *Ciufolini & Wheeler (1995), Ch.7*

- *"An eminent physicist spent the last years of his life pushing the idea that gravitation follows the pattern of EM. This thesis we cannot accept, and the community of physics, quite rightly, does not accept"*
- *"Nonetheless there is an important lesson about gravity by treating it on the incorrect basis that it behaves like EM"*
  - The "Sciama sum for inertia", $\sum_i \frac{M_i}{r_i}\frac{G}{c^2} \sim 1$
- The "voting power" M_i/r_i follows from the Kerr metric in the far field limit:

$$ds^2 = -(1-2M/r)dt^2 + (1+2M/r)dr^2 + r^2 d\Omega - 4(J/r)\sin^2\theta\, d\phi\, dt$$

- *"What can we conclude from this analysis? First, nothing provable. Without using a sound theory of gravity, we cannot expect a soundly founded theory of gravitational radiative reaction [inertia]."*

Technical session:
The Mirage of Maxwellian Gravity

With the advantages of history and modern textbooks, we can now enumerate the reasons Maxwellian gravity vanishes in a mirage:

- Gauge freedom lets you see whatever you want, but it's not all real
- Linear theory does not provide exchange of gravitational energy or momentum with matter
- Maxwellian potentials are not radiative

The force equation in GR

$$\nabla_\mu T^{\mu\nu} = \frac{\partial T^{\mu\nu}}{\partial x^\mu} + \Gamma^\mu_{\mu\alpha} T^{\alpha\nu} + \Gamma^\nu_{\mu\alpha} T^{\mu\alpha} = 0$$

If $T^{\mu\nu} = \rho U^\mu U^\nu \Rightarrow \frac{dU^\mu}{d\tau} + \Gamma^\mu_{\alpha\beta} U^\alpha U^\beta = 0$

This is the general force equation for any inertial induction device.
A calculation starts here.

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The linear force equation in GR

(following Carroll sec. 7.2)

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2) \qquad h_{\mu\nu} \Rightarrow h_{00} \equiv -2\Phi, \; h_{0i} \equiv A^i$$

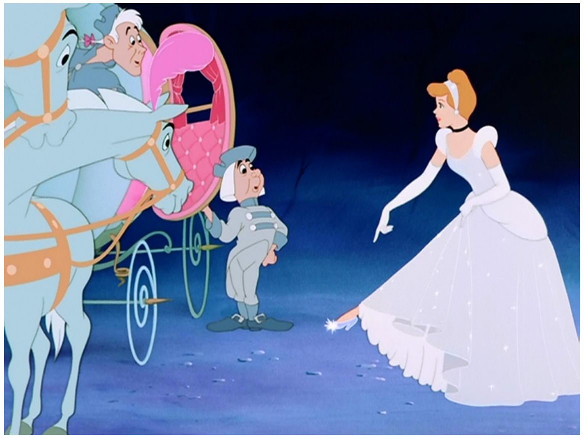
$$\frac{dU^\mu}{d\tau} + \Gamma^\mu_{\alpha\beta} U^\alpha U^\beta = 0 \qquad p^\mu = mU^\mu \;, \; \boldsymbol{p} = p^t \boldsymbol{v}$$

$$\frac{d\boldsymbol{p}}{dt} = p^t [\boldsymbol{E} + (\boldsymbol{v} \times \boldsymbol{B})] - 2 \frac{\partial h_{ij}}{\partial t} v^j - \left(\frac{\partial h_{ki}}{\partial x^j} + \frac{\partial h_{ji}}{\partial x^k} - \frac{1}{2} \frac{\partial h_{jk}}{\partial x^i} \right) v^j v^k$$

$$\boldsymbol{E} \equiv -\nabla \Phi - \frac{\partial \boldsymbol{A}}{\partial t} \qquad \boldsymbol{B} \equiv \nabla \times \boldsymbol{A}$$

The linear geodesic equation has effects similar to the Lorentz force law of EM.
The time-time component of h behaves like an electric potential,
and the time-space components of h behave like the a magnetic vector potential.
E and **B** are gravitoelectric and gravitomagnetic fields.

- The effect of weak gravitation on matter includes EM-like effects from the time components of the metric perturbation, which behave like the electromagnetic potential
- So far, linear GR is looking Maxwellian.
- We have our glass slippers and are off to the Maxwellian ball!



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Not so fast, Cinderella

- You have found the gravitomagnetic force to first order in the geodesic equation
- But to first order in the Einstein field equations, the matter obeys simple conservation of energy
  - Carroll p.276, Weinberg 10.1.6, Poisson & Will 5.135

$$\frac{\partial T^{\mu\nu}}{\partial x^{\mu}} = 0$$

- There is no interaction between matter and the gravitational field in linear GR. The field equations must be carried to second order to investigate coupling between matter and gravitational fields

Inertial induction, if it exists, cannot be described by linear GR, Maxwellian or otherwise, because there is no exchange of gravitational energy or momentum with matter to this order.

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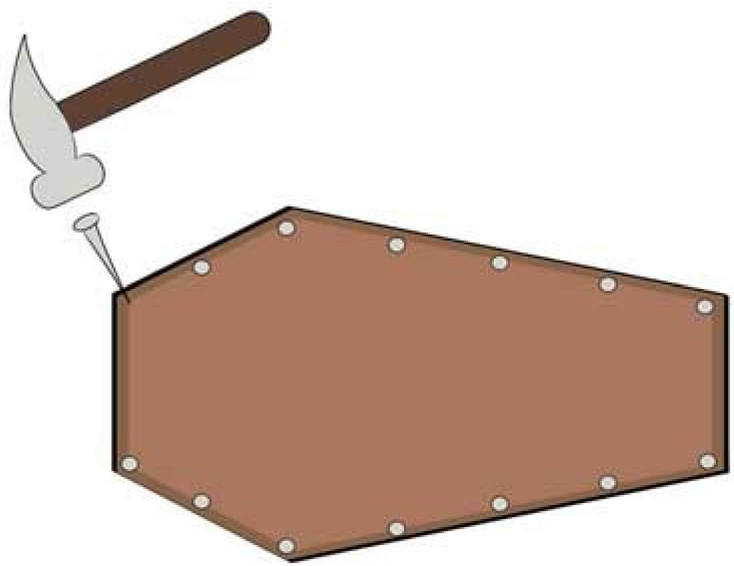
This is the nail in the coffin of Sciama's dream to explain inertia from Maxwellian gravity



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But aren't the field equations Maxwellian to linear order?

- Yes and no. You can find Maxwellian field equations in GR, but it's a mirage due to a poor choice of coordinate system.
- When you look clearly at the degrees of freedom in the gravitational field, the Maxwellian mirage evaporates
- Gauge freedom makes GR into a hall of mirrors

Gauge Freedom

Maxwell equations

$$\partial_\mu F^{\mu\nu} = 4\pi J^\nu$$

4 eqns in 4 unknowns, A^μ

However, from conservation of charge

$$\partial_\nu J^\nu = 0 = \partial_{\mu\nu} F^{\mu\nu}$$

Therefore the 4 Maxwell eqns are not independent. They obey one constraint eqn among the 4 unknowns. Therefore they provide only 3 eqns in 4 unknowns.

The fourth eqn in the 4 unknowns comes from the choice of gauge. It is one free equation among the 4 unknowns, typically chosen to make life easier. There are many EM gauges to choose from.

Gauge is only a problem when dealing with potentials. It does not affect the force eqns.

Einstein equations

$$R_{\mu\nu} - R\,g_{\mu\nu}/2 = 8\pi T_{\mu\nu}$$

10 eqns in 10 unknowns, $g_{\mu\nu}$

However, from conservation of energy/momentum

$$\nabla^\mu T_{\mu\nu} = 0 = \nabla^\mu \left(R_{\mu\nu} - R g_{\mu\nu}/2\right)$$

Therefore the 10 Einstein eqns are not independent. They obey 4 constraint eqns among the 10 unknowns. Therefore they provide only 6 eqns in 10 unknowns.

The extra 4 eqns in the 10 unknowns comes from the choice of coordinates. They are 4 free eqns among the 10 unknowns, typically chosen to make life easier. This is gauge freedom in GR.

In EM, 25% of the field eqns are arbitrary.
In GR, 40% of the field eqns are arbitrary.

You can see whatever you want in the field eqns, but it's not all real

Linearized Field Equations

(grav radiation, precession of Mercury, deflection of starlight)

When $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2)$

$$R_{\mu\nu} = \frac{1}{2}\left(\partial_\sigma \partial_\nu h^\sigma_\mu + \partial_\sigma \partial_\mu h^\sigma_\nu - \partial_\mu \partial_\nu h - \partial_\alpha \partial^\alpha h_{\mu\nu}\right) + O(h^2)$$

(Weinberg 10.1.4)

Choose harmonic gauge,
aka Lorenz gauge $\partial_\mu h^\mu_\nu = \frac{1}{2}\partial_\nu h^\mu_\mu$

$(\partial_\alpha \partial^\alpha = \nabla^2 - \partial_{tt})$

Then $R_{\mu\nu} = -\partial_\alpha \partial^\alpha h_{\mu\nu} = 16\pi G\left(T_{\mu\nu} - \eta_{\mu\nu} T^\alpha_\alpha\right)$

$(\partial_\alpha \partial^\alpha A^\mu = 4\pi J^\mu)$

A wave equation for each component of h!
This looks Maxwellian. But it’s an illusion of the gauge

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What is real and what is gauge?

- With 4 of the 10 field equations freely chosen, care is needed to
  - isolate the 6 “true” gravitational potentials
  - deduce the nature of the “true” gravitational field
  - avoid the mirages of a poor gauge choice
- We do this by considering gauge-invariant potentials
  - the true 6 degrees of freedom of the grav field
  - “coulomb gauge” (Poisson & Will) or “transverse gauge” (Carroll)
  - Poisson & Will 5.5.5

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Blue pill or red pill?

The true nature of the linear gravitational field

$$R_{\mu\nu} - R\,g_{\mu\nu}/2 \;=\; 8\pi G\,T_{\mu\nu} \qquad g_{\mu\nu} \;=\; \eta_{\mu\nu} + h_{\mu\nu} + O(h^2)$$

$$h_{\mu\nu} \;\Rightarrow\; h_{00} \equiv \Phi,\;\; h_{0i} \equiv A^i,\;\; \psi \equiv \delta^{ij} h_{ij},\;\; h_{ij} \equiv \psi\,\delta_{ij} + h_{ij}^{TF},\;\; \delta^{ij} h_{ij}^{TF} = 0$$

Potential-invariant gauge (4 eqns) $\partial_i h_{TF}^{ij} = 0,\;\; \partial_i A^i = 0$

True degrees of freedom: $\phi,\;\; \psi,\;\; 2\,\mathrm{of}\,\mathrm{A}^{\mathrm{i}},\;\; 2\,\mathrm{of}\,\mathrm{h}_{\mathrm{ij}}^{\mathrm{TF}}$

Gauge-invariant linear field equations

(Poisson & Will 5.5.5)

$$\nabla^2 \psi \;\propto\; G\,T_{00} \qquad \nabla^2 A_i \;\propto\; G\,T_{0i}$$

$$\nabla^2 (\Phi - \psi) \;\propto\; G\,T_{ij}\,\delta^{ij} \qquad (\nabla^2 - \partial_{tt})\, h_{ij}^{TF} \;\propto\; G\,T_{ij}^{TF}$$

$$(\nabla^2 \equiv \partial_i \partial^i)$$

In EM, all 4 potentials are radiative, and 3 are true degrees of freedom.
In gravity, only 6 of the 10 potentials are radiative; of those, only 2 are true degrees of freedom.
The potentials that produce Maxwellian forces in the linear geodesic equation are not radiative.

Maxwellian Gravity, R.I.P.

- The linear geodesic equation produces gravitoelectric and gravitomagnetic forces, arising from the components h_{0j} of the metric perturbation, very similar to the EM vector potential. But that is where the similarity to EM stops.
- Hopes for a Maxwellian limit of GR are dashed because
  - There is no exchange of energy and momentum between matter and gravitational fields in linear GR. Effects of the gravitoelectric and gravitomagnetic fields require second order field equations.
    - Inertial induction cannot be treated in linear GR
  - The "true" gauge-invariant gravitational potentials in the linear theory are not Maxwellian, they are sui generis.
  - the Maxwellian-like components h_{0j} in the force equation are non-radiative. Of the 10 gravitational potentials, only 2 have true radiative degrees of freedom.

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How to Bring Inertial Induction in from the Cold

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backup

3 4, 153 (1971).
[2]P. G. Burke and W. D. Robb, J. Phys. B 5, 44 (1972).
[3]M. J. Seaton, J. Phys. B 7, 1817 (1974).
[4]F. E. Harris and H. H. Michels, Phys. Rev. Lett. 22, 1036 (1969).
[5]R. K. Nesbet, Phys. Rev. 179, 60 (1969).
[6]G. J. Seiler, R. S. Oberoi, and J. C. Callaway, Phys. Rev. A 3, 2006 (1963).
[7]E. R. Smith and R. J. W. Henry, Phys. Rev. A 7, 1585 (1973).
[8]E. R. Smith and R. J. W. Henry, Phys. Rev. A 8, 572 (1973).
[9]N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Clarendon, Oxford, 1965), p. 530.

Absence of Inertial Induction in General Relativity

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(Received 24 June 1977)

I review arguments indicating that there is no real, physically detectable, local inertial-induction effect in general relativity, contrary to recent comments by Tittle.

In a recent Letter Tittle[1] has brought up an old suggestion of Einstein's that there is some sort of inertial-induction effect in his standard general-relativistic theory of gravitation. In his book Einstein[2] devoted about ten pages to a discussion of this point, particularly in reference to the role of Mach's principle in his theory. Over the years, many and varied expressions of Mach's principle have been proposed, making it one of the most elusive concepts in physics.[3] However, it seems clear that Einstein intended to show that locally measured inertial-mass values are gravitationally coupled to the mass distribution in the universe in his theory. For convenience I repeat the first-order geodesic equations given by Einstein to support his argument:

$$(d/dl)[(l+\bar{\sigma})\vec{v}] = \nabla\bar{\sigma} + \partial\vec{A}/\partial l + \nabla\times(\vec{A}\times\vec{v}),$$

$$\bar{\sigma} = (\kappa/8\pi)\int(\sigma/r)\,dV_0,$$

$$\vec{A} = (\kappa/2\pi)\int(\sigma d\vec{x}/dl)\,r^{-1}\,dV_0.$$

Here σ is the source-mass density while l is *coordinate* time and $\vec{v}$ is *coordinate* velocity of a test particle. Einstein's claim is that "The inertial mass is proportional to $l+\bar{\sigma}$, and therefore increases when ponderable masses approach the test body."[2] This Letter is meant to call attention to the problem, Dicke and I were not satisfied that general relativity met this criterion. In fact, we came to the conclusion that Einstein's claim of inertial induction was a purely coordinate effect and thus could have no physically detectable consequences. The basic reason is that Einstein's theory is generally covariant, with gravitational effects carried by a tensor field alone whose effects are transformed away approximately in any local inertial reference frame. We neglect, of course, tidal forces which have no significant effect in a cosmological context. Because of the central importance of this problem, I have given a careful and thorough treatment of it.[4,5] Since Tittle, and perhaps others, do not seem to be aware of this work, a review of the main points of the argument will be given here.

First, let us recall the importance of giving operational definitions for our terms, as stressed by Einstein, above all. Thus the concept of inertia must be tied to some, at least ideally possible, measurement. Of course, mass is a dimensional quantity, so we must pick some standard unit. Since there does not seem to be any direct connection (pending development of a complete unified field theory) between small electrical and other atomic and nuclear fields and gravitational

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Mach's Principle and the Locally Measured Gravitational Constant in General Relativity*

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(Received August 14, 1961)

It has been conjectured that a "Mach's principle" might lead to a dependence of the local Newtonian gravitational constant, K, on universe structure, $K^{-1}{\sim}M/R$. Einstein and others have suggested that general relativity predicts such a result. A closer analysis, however, including the carrying out of the geodesic equations to second order, seems to indicate that this is not true and that the apparent "Mach's principle" terms involving total universe structure are really only coordinate effects. Further, the measure of gravitating mass obtained in a local, proper Newtonian gravitational experiment is compared in a coordinate-free way to an experimentally measurable inertial mass and found to be related to it in a way independent of the rest of the universe. A generalization of these results is given. It is based on the fact that in general relativity the only way the universe can influence experiments done in an electrically shielded laboratory is through the metric and that this can be "transformed away" to any degree of accuracy for a sufficiently small laboratory. Consequences of this are summarized in Dicke's "strong principle of equivalence." It is noted, however, that there are other statements which might be called "Mach's principles" which are satisfied in general relativity.

I. INTRODUCTION

THE principal idea which guided Einstein in

where A is a constant dimensionless number. For a more general type of universe with masses m_a at distances r_a from some point x, this might be extended to