Pursuit of Inertial Induction:

Propellantless Propulsion
within the
Known Laws of Gravity

Lance Williams
Konfluence Research

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Context: General Relativity *Triumphant*

- A breakthrough prospect in an area with an active, established base
  - We can stand on the shoulders of giants here
  - Man-centuries of theoretical and experimental experience in this very question
- No prospect for inertial induction outside GR
- No prospect for a breakthrough in GR unknown to gravity researchers
- APS DGRAV ~~can~~ must guide us and manage our ignorance
  - Peers who understand the conceptual basis
  - Conceived and detected gravitational waves; and dark energy; and dark matter
  - A condition for "responsible" funding; no "hail mary"s
    - In the absence of someone who levitates a cannonball

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What is Inertial Induction?

- *Local* gravitational exchange of momentum with the *distant* universe
- The basic principle, if it exists, underlying propellantless propulsion advances within the framework of the accepted theory of gravity, general relativity
- "inertial induction" is the historical generic term referring to mechanical coupling with the distant universe
  - Used in the Brans 1977 paper on this topic
  - suggested as preferable to "Machian effects" and "Mach's principle", in the 1995 Barbour & Pfister proceedings on Mach's Principle

Mainstream View of Inertial Induction

- Einstein abandoned Mach's Principle when he realized the field equations allow a spacetime metric to emerge from a universe empty of matter
  - cf. *Subtle is the Lord*, by Abraham Pais, p. 287
- The modern view is that inertial induction violates the Equivalence Principle
  - The gravitational field of the universe is not measurable in free fall

The Equivalence Principle

- The conceptual cornerstone of general relativity
  - Einstein's "happiest thought"
- Gravity vanishes in free fall
  - There is gravity at the space station. It holds the station in its orbit! But the astronauts are "weightless" there
  - How can you couple to the distant universe if its gravity vanishes in free fall or in deep space far from any object?
- Modern physics recognizes several flavors
  - The Weak EP
  - The Strong EP
  - The Einstein EP

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Absence of Inertial Induction in General Relativity

Carl H. Brans
Physics Department, Loyola University, New Orleans, Louisiana 70118
(Received 24 June 1977)

I review arguments indicating that there is no real, physically detectable, local inertial-induction effect in general relativity, contrary to recent comments by Tittle.

In a recent Letter Tittle[1] has brought up an old suggestion of Einstein's that there is some sort of inertial-induction effect in his standard general-relativistic theory of gravitation. In his book Einstein[2] devoted about ten pages to a discussion of this point, particularly in reference to the role of Mach's principle in his theory. Over the years, many and varied expressions of Mach's principle have been proposed, making it one of the most elusive concepts in physics.[3] However, it seems clear that Einstein intended to show that locally measured inertial-mass values are gravitationally coupled to the mass distribution in the universe in his theory. For convenience I repeat the first-order geodesic equations given by Einstein to support his argument:

$$(d/dl)[(l+\bar{\sigma})\vec{v}] = \nabla\bar{\sigma} + \partial\vec{A}/\partial l + \nabla\times(\vec{A}\times\vec{v}),$$

$$\bar{\sigma} = (\kappa/8\pi)\int(\sigma/r)\,dV_0,$$

$$\vec{A} = (\kappa/2\pi)\int(\sigma d\vec{x}/dl)\,r^{-1}\,dV_0.$$

Here σ is the source-mass density while l is *coordinate* time and $\vec{v}$ is *coordinate* velocity of a test particle. Einstein's claim is that "The inertial mass is proportional to $l+\bar{\sigma}$, and therefore increases when ponderable masses approach the test body."[2] This Letter is meant to call attention to the fact that, as a result of a close study of the problem, Dicke and I were not satisfied that general relativity met this criterion. In fact, we came to the conclusion that Einstein's claim of inertial induction was a purely coordinate effect and thus could have no physically detectable consequences. The basic reason is that Einstein's theory is generally covariant, with gravitational effects carried by a tensor field alone whose effects are transformed away approximately in any local inertial reference frame. We neglect, of course, tidal forces which have no significant effect in a cosmological context. Because of the central importance of this problem, I have given a careful and thorough treatment of it.[4,5] Since Tittle, and perhaps others, do not seem to be aware of this work, a review of the main points of the argument will be given here.

First, let us recall the importance of giving operational definitions for our terms, as stressed by Einstein, above all. Thus the concept of inertia must be tied to some, at least ideally possible, measurement. Of course, mass is a dimensional quantity, so we must pick some standard unit. Since there does not seem to be any direct connection (pending development of a complete unified field theory) between small electrical and other atomic and nuclear fields and gravitational

Experimental Tests of Inertial Induction

- Clifford Will has pioneered many tests of GR and the equivalence principle
  - cf. Barbour & Pfister, 1995, p365
- He identifies 3 categories of "Machian effects"
  - Weak EP violations => ruled out
  - Strong EP violations => ruled out
  - Gravitomagnetism => NOT ruled out

Why Inertial Induction is Viable Yet

- Equivalence of inertial and gravitational mass
  - An unexplained coincidence going back to Newton
  - Implies inertia has a link to gravity
- Gravitational potential of the universe is similar to the speed of light squared
  - Gravitational potential energy from the universe is similar to rest mass energy
  - An unexplained coincidence going back to the 1950s
- Inertia manifests upon (is “induced” by) acceleration
  - Acceleration does not obey the EP
  - Free fall is an inertial frame, acceleration is not
- A toy electromagnetic model of gravity describes inertial induction
  - Sciama 1953
  - Ciufolini & Wheeler, 1995, argue the result is also in GR

Gravity is nothing like electromagnetism

Gravity	Electromagnetism
Different masses fall the same	Different charges fall differently
Spacetime is dynamic	Spacetime is static
$\frac{d^2 x^\mu}{d\tau^2} \propto \frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}$	$\frac{d^2 x^\mu}{d\tau^2} \propto \frac{dx^\nu}{d\tau}$
Field equations non-linear	Field equations linear
quadropole	dipole
10 components of the gravitational potential	4 components of the electromagnetic potential
Grav fields feel grav, Like everything else	EM fields don't feel EM (EM fields aren't charged)

Wheeler vs Sciama

- Sciama, 1953
  - Assume gravity obeys Maxwell equations of electromagnetism, *not* the Einstein equations
  - Flat universe => inertia can be understood as inductive effect
- Ciufolini & Wheeler, 1995
  - Compelled by Sciama's result
  - Recognize the indefensibility of Sciama's assumption
  - Argue the basic result still pertains within GR
    - "mass there determines inertia here"

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Ciufolini & Wheeler (1995), Ch.7

- *"An eminent physicist spent the last years of his life pushing the idea that gravitation follows the pattern of EM. This thesis we cannot accept, and the community of physics, quite rightly, does not accept"*
- *"Nonetheless there is an important lesson about gravity by treating it on the incorrect basis that it behaves like EM"*
  - The "Sciama sum for inertia", $\sum_i \frac{M_i}{r_i}\frac{G}{c^2} \sim 1$
- The "voting power" M_i/r_i follows from the Kerr metric in the far field limit:

$$ds^2 = -(1-2M/r)dt^2 + (1+2M/r)dr^2 + r^2 d\Omega - 4(J/r)\sin^2\theta\, d\phi\, dt$$

- *"What can we conclude from this analysis? First, nothing provable. Without using a sound theory of gravity, we cannot expect a soundly founded theory of gravitational radiative reaction [inertia]."*

The Laws of Gravity

How gravity responds to matter (field equations)

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi \frac{G}{c^4} T_{\mu\nu}$$

How matter responds to gravity (force equation)

$$\nabla_\mu T^{\mu\nu} = 0$$

- The equations have passed numerous experimental tests
- No situation has gone unexplained by these equations
- However: the field equations are difficult. 10 coupled, second-order, non-linear (quadratic) partial differential equations
  - Exact analytic solutions are as rare as hen's teeth, with high symmetry only

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The force equation in GR

$$\nabla_\mu T^{\mu\nu} = \frac{\partial T^{\mu\nu}}{\partial x^\mu} + \Gamma^\mu_{\mu\alpha} T^{\alpha\nu} + \Gamma^\nu_{\mu\alpha} T^{\mu\alpha} = 0$$

If $T^{\mu\nu} = \rho U^\mu U^\nu \Rightarrow \frac{dU^\mu}{d\tau} + \Gamma^\mu_{\alpha\beta} U^\alpha U^\beta = 0$

This is the general force equation for any inertial induction device.
A calculation starts here.

Complexity of Grav Forces

$$\Gamma^{\mu}_{\alpha\beta}$$

- Gravitational force has 40 components
- Only 6 are named or intuitively comprehended
  - 3 are Newtonian gravity (gravitoelectric)
  - 3 are gravitomagnetic
- There may be gravitational force effects yet unexplored

The linear force equation in GR

(following Carroll sec. 7.2)

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2) \qquad h_{\mu\nu} \Rightarrow h_{00} \equiv -2\Phi, \quad h_{0i} \equiv A^i$$

$$\frac{dU^\mu}{d\tau} + \Gamma^\mu_{\alpha\beta} U^\alpha U^\beta = 0 \qquad p^\mu = mU^\mu \;, \quad \boldsymbol{p} = p^t \boldsymbol{v}$$

$$\frac{d\boldsymbol{p}}{dt} = p^t [\boldsymbol{E} + (\boldsymbol{v} \times \boldsymbol{B})] - 2\frac{\partial h_{ij}}{\partial t} v^j - \left(\frac{\partial h_{ki}}{\partial x^j} + \frac{\partial h_{ji}}{\partial x^k} - \frac{1}{2}\frac{\partial h_{jk}}{\partial x^i} \right) v^j v^k$$

$$\boldsymbol{E} \equiv -\nabla\Phi - \frac{\partial \boldsymbol{A}}{\partial t} \qquad \boldsymbol{B} \equiv \nabla \times \boldsymbol{A}$$

The linear geodesic equation has effects similar to the Lorentz force law of EM.
The time-time component of h behaves like an electric potential,
and the time-space components of h behave like the a magnetic vector potential.
E and **B** are gravitoelectric and gravitomagnetic fields.

Gravitomagnetism

- AKA Lens-Thirring effect
- AKA frame-dragging
- Only treated in circular configurations
- No linear accelerations considered
  - Inertia is an effect of linear acceleration
- Not ruled out by Cliff Will as a coupling effect with the distant universe

Pitfalls of Inertial Induction in GR

- Gauge freedom
  - 40% of the field equations are arbitrary, arising from the choice of coordinates
  - You can see mirages in GR if you're not careful
  - You can find EM-like field equations in GR, but they're not real
- Linear theory
  - Makes GR analytically tractable
  - Only works for weak gravity
  - No exchange of momentum between gravity and matter
- Maxwellian gravity cannot describe coupling with the universe

Gauge Freedom

Maxwell equations

$$\partial_\mu F^{\mu\nu} = 4\pi J^\nu$$

4 eqns in 4 unknowns,

However, from conservation of charge

$$\partial_\nu J^\nu = 0 = \partial_{\mu\nu} F^{\mu\nu}$$

Therefore the 4 Maxwell eqns are not independent. They obey one constraint eqn among the 4 unknowns. Therefore they provide only 3 eqns in 4 unknowns.

The fourth eqn in the 4 unknowns comes from the choice of gauge. It is one free equation among the 4 unknowns, typically chosen to make life easier. There are many EM gauges to choose from.

Gauge is only an issue when dealing with potentials. It does not affect the force eqns.

Einstein equations

$$R_{\mu\nu} - R\, g_{\mu\nu}/2 = 8\pi T_{\mu\nu}$$

10 eqns in 10 unknowns,

However, from conservation of energy/momentum

$$\nabla^\mu T_{\mu\nu} = 0 = \nabla^\mu (R_{\mu\nu} - R g_{\mu\nu}/2)$$

Therefore the 10 Einstein eqns are not independent. They obey 4 constraint eqns among the 10 unknowns. Therefore they provide only 6 eqns in 10 unknowns.

The extra 4 eqns in the 10 unknowns comes from the choice of coordinates. They are 4 free eqns among the 10 unknowns, typically chosen to make life easier. This is gauge freedom in GR.

In EM, 25% of the field eqns are arbitrary.
In GR, 40% of the field eqns are arbitrary.

The true nature of the linear gravitational field

$$R_{\mu\nu} - R g_{\mu\nu}/2 \;=\; 8\pi G T_{\mu\nu} \qquad g_{\mu\nu} \;=\; \eta_{\mu\nu} + h_{\mu\nu} + O(h^2)$$

$$h_{\mu\nu} \;\Rightarrow\; h_{00} \equiv \Phi,\;\; h_{0i} \equiv A^i,\;\; \psi \equiv \delta^{ij} h_{ij},\;\; h_{ij} \equiv \psi\,\delta_{ij} + h_{ij}^{TF},\;\; \delta^{ij} h_{ij}^{TF} = 0$$

Potential-invariant gauge (4 eqns) $\partial_i h_{TF}^{ij} = 0,\;\; \partial_i A^i = 0$

True degrees of freedom: $\phi,\;\; \psi,\;\; 2\,\mathrm{of}\;\mathrm{A}^{\mathrm{i}},\;\; 2\,\mathrm{of}\;\mathrm{h}_{\mathrm{ij}}^{\mathrm{TF}}$

Gauge-invariant linear field equations

(Poisson & Will 5.5.5)

$$\nabla^2 \psi \;\propto\; G T_{00} \qquad \nabla^2 A_i \;\propto\; G T_{0i}$$

$$\nabla^2 (\Phi - \psi) \;\propto\; G T_{ij} \delta^{ij} \qquad (\nabla^2 - \partial_{tt}) h_{ij}^{TF} \;\propto\; G T_{ij}^{TF}$$

$$(\nabla^2 \equiv \partial_i \partial^i)$$

In EM, all 4 potentials are radiative, and 3 are true degrees of freedom.
In gravity, only 6 of the 10 potentials are radiative; of those, only 2 are true degrees of freedom.
The potentials that produce Maxwellian forces in the linear geodesic equation are not radiative.

Next Steps to Pursue Inertial Induction

- Solve the full, non-linear field equations with appropriate boundary conditions
  - Allow exchange of momentum with gravitational field
  - The gravitational field of the universe is not small
- Evaluate the force equation for the case of linear acceleration
  - Literature search on linear gravitomagnetism
  - Impervious to Equivalence Principle concerns
- Find testable experimental configurations from the theory
- If feasible experiments exist, execute them
- Present results to skeptical peers
  - American Physical Society – DGRAV
  - Peer-reviewed publications – Physical Review D