A Propulsion Breakthrough within the Known Laws of Gravity: ELECTROMAGNETIC GRAVITY CONTROL

Kaluza 5D theory in a nutshell

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Vector and tensor notation

- Greek indices span 4 dimensions of spacetime.
- Roman indices span 5 dimensions.
- 5D tensors are written with a tilde

ϕ → a 4D scalar with one field component

A^{μ} → a 4D vector with 4 components

$\tilde{U}^{a}$ → a 5D vector with 5 components

$\tilde{U}_{5}$ → the 5th component of a covariant 5D vector

$g_{\mu\nu}$ → a 4D symmetric tensor with 10 components

$\tilde{g}_{ab}$ → a 5D symmetric tensor with 15 components

$\tilde{g}_{\mu\nu}$ → the spacetime components of a 5D symmetric tensor

Kaluza Field Equations

$$\tilde{g}_{ab} \sim \left(\begin{array}{c|c} g_{\mu\nu} & k A_\mu \\ \hline k A_\nu & \phi^2 \end{array} \right) \qquad \tilde{G}_{ab} = \frac{8\pi G}{c^4} \tilde{T}_{ab}$$

$$\partial \tilde{g}_{ab} / \partial x^5 = 0 \qquad k = \sqrt{16\pi G \epsilon_0 / c^2} = 5.8 \times 10^{-19} \, \text{mks}$$

$$\left(\begin{array}{c|c} G_{\mu\nu} - T^{\phi}_{\mu\nu} - \frac{k^2}{2} \phi^2 T^{EM}_{\mu\nu} & k \nabla^\nu (\phi^3 F_{\mu\nu}) \\ \hline k \nabla^\mu (\phi^3 F_{\nu\mu}) & \frac{3}{4} k^2 \phi^4 F_{\mu\nu} F^{\mu\nu} - \phi^2 R \end{array} \right) \sim \left(\begin{array}{c|c} G T^{M}_{\mu\nu} & k \mu_0 J_\mu \\ \hline k \mu_0 J_\nu & G \tilde{T}_{55} \end{array} \right)$$

Kaluza Field Lagrangian

$$L = \tilde{g}^{1/2}\tilde{R} = g^{1/2}\left[\phi \frac{g^{\mu\nu}R_{\mu\nu}}{16\pi G} - \frac{1}{4\mu_0}\phi^3 g^{\alpha\mu} g^{\beta\nu} F_{\alpha\beta} F_{\mu\nu}\right]$$

- Ferrari (1989)
- Coquereaux & Esposito-Farese (1990)
- Williams (2015)

- there are no derivatives of the KSF, so its field equation is algebraic
- the KSF acts simultaneously as a variable gravitational constant, and as a variable electric constant
- a conformal transformation removes the variable gravitational constant, and adds a term in KSF derivatives, but particles still move on geodesics of $g_{\mu\nu}$
- this looks like Brans-Dicke "$\omega=0$", but that limit is not appropriate here. Standard GR results when $\Phi \rightarrow 1$

5D Geodesic Motion

$$\tilde{U}^a \tilde{\nabla}_a \tilde{U}^b = 0$$

$$\tilde{U}^a \equiv \frac{dx^a}{ds} \qquad ds^2 \equiv \tilde{g}_{ab}\, dx^a dx^b$$

$$\frac{\partial \tilde{g}_{ab}}{\partial x^5} = 0 \rightarrow \tilde{g}_{5a} \tilde{U}^a \equiv \boxed{\tilde{U}_5 = \phi^2 \left(\tilde{U}^5 + kA_\mu \tilde{U}^\mu \right) = \text{constant}}$$

the cylinder condition imposes a non-trivial constant of the motion along the fifth coordinate

$$= \phi^2 \left(\frac{d\tau}{ds} \right) \left(U^5 + kA_\mu U^\mu \right) \equiv \phi^2 \left(\frac{d\tau}{ds} \right) U_5$$

$$c^2 d\tau^2 \equiv g_{ab}\, dx^\mu dx^\nu \qquad ds^2 = c^2 d\tau^2 + \phi^2 \left(dx^5 + kA_\nu dx^\nu \right)^2$$

Kaluza coupling coefficient

$$\boxed{\frac{c\, d\tau}{ds} = \sqrt{1 - \tilde{U}_5^2 / \phi^2}}$$

note that $\tilde{U}_5 / \phi \leq 1$

4D Equation of Motion

and identification of electric charge

spacetime components of 5D geodesic

$$\frac{d\tilde{U}^{\nu}}{ds} + \tilde{\Gamma}^{\nu}_{bc}\tilde{U}^{b}\tilde{U}^{c} = 0$$

$$\frac{d\tilde{U}^{\nu}}{ds} + \Gamma^{\nu}_{\alpha\beta}\tilde{U}^{\alpha}\tilde{U}^{\beta} + kg^{\mu\nu}\tilde{U}_{5}\tilde{U}^{\alpha}F_{\alpha\mu} - g^{\nu\alpha}\phi(\partial_{\alpha}\phi)(\tilde{U}_{5}/\phi^{2})^{2} = 0$$

- Ferrari (1989)
- Coquereaux & Esposito-Farese (1990)
- Gegenberg & Kunstatter (1984)

electric charge

scalar charge

$$\frac{dU^{\nu}}{d\tau} + \Gamma^{\nu}_{\alpha\beta}U^{\alpha}U^{\beta} = kU_{5}g^{\mu\nu}U^{\alpha}\phi^{2}F_{\mu\alpha} + U_{5}^{2}\phi(\partial_{\alpha}\phi)[g^{\nu\alpha} - U^{\nu}U^{\alpha}/c^{2}]$$

gravity

electromagnetism

scalar force

identification of electric charge

$$kU_{5} = k(U^{5} + kA_{\mu}U^{\mu}) \rightarrow Q/m$$

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Aspects of the Kaluza scalar force

- simultaneously a variable gravitational constant and a variable electric constant
- truly a "fifth force"
  - completes the trinity of forces: tensor, vector, and scalar
- long-range
- boson must be massless, spin 0
  - the photon is spin 1
  - the graviton should be spin 2, but may never be detected
- scalar charge is quadratic in electric charge
  - the KSF term in the force equation is potentially enormous compared to the electric force, but KSF gradients are always small so the term vanishes
- electric charge is required for a test body to couple to the KSF force, yet the KSF can be generated indirectly by neutral matter and by its own self energy
- EM energy-momentum is uniquely transparent to the KSF, but the KSF requires or accompanies bulk EM fields (not radiation)

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5D Energy-Momentum Tensor and Lagrangian for Charged Dust/Cold Fluid

- The geodesic equation holds only for non-interacting particles ("dust"). We therefore complete Kaluza's 5D field equations with a properly covariant source term for 5D dust

$$\tilde{G}_{ab} = \frac{8\pi G}{c^3} \frac{\tilde{\rho}}{\tilde{g}^{1/2}} \frac{\tilde{U}_a \tilde{U}_b}{(dt/ds)}$$

Energy-momentum tensor of 5D dust

$$\tilde{U}^a \equiv \frac{dx^a}{ds}$$

$$ds^2 \equiv \tilde{g}_{ab}\, dx^a\, dx^b$$

Lagrangian of 5D dust

$$\tilde{S}_M = \tilde{g}^{1/2} \tilde{L}_M = -\frac{1}{2} \frac{\tilde{\rho}\, c\, \tilde{U}_a \tilde{U}_b}{(dt/ds)} \tilde{g}^{ab}$$

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Field Equations with Sources

$$\tilde{G}_{ab} = \frac{8\pi G}{c^4}\tilde{T}_{ab} \rightarrow \frac{8\pi G}{c^3}\frac{\tilde{\rho}}{\phi\, g^{1/2}}\tilde{g}_{ac}\frac{dx^c}{dt}\tilde{g}_{bd}\frac{dx^d}{ds}$$

Einstein equations, modified with a scalar field

$$G_{\mu\nu} = T^{\phi}_{\mu\nu} + \frac{8\pi G}{\mu_0 c^4}\phi^2 T^{EM}_{\mu\nu} + \frac{8\pi G}{c^3}\frac{1}{\phi}\frac{d\tau}{ds}\frac{\tilde{\rho}}{g^{1/2}} g_{\mu\alpha}\frac{dx^\alpha}{dt} g_{\nu\beta} U^\beta$$

Maxwell equations, modified with a scalar field

$$\nabla^\alpha\left(\phi^3 F_{\nu\alpha}\right) = \mu_0 k c\, \tilde{U}_5 \frac{\tilde{\rho}}{g^{1/2}} g_{\nu\mu}\frac{dx^\mu}{dt}$$

charge density $\sigma \equiv \tilde{\rho}\, Q/m$

Scalar field equation

$$\phi^2\left[\frac{3}{4}k^2\phi^2 F_{\alpha\beta}F^{\alpha\beta} - R\right] = \mu_0 k \frac{\sigma}{g^{1/2}}\phi\frac{d\tau}{dt}\tilde{U}_5$$

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