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Presence of inertial induction in general relativity

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Absence of Inertial Induction in General Relativity

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I review arguments indicating that there is no real, physically detectable, local inertial-induction effect in general relativity, contrary to recent comments by Tittle.

In a recent Letter Tittle[1] has brought up an old suggestion of Einstein's that there is some sort of inertial-induction effect in his standard general-relativistic theory of gravitation. In his book Einstein[2] devoted about ten pages to a discussion

lem, Dicke and I were not satisfied that general relativity met this criterion. In fact, we came to the conclusion that Einstein's claim of inertial induction was a purely coordinate effect and thus could have no physically detectable consequences

What is inertia?

It takes work to accelerate a massive object. What is pushing back?

$$\boldsymbol{F} = M_i \frac{d\boldsymbol{v}}{dt}$$

acceleration = (applied force) / (inertial mass)

Inertia involves a local momentum transfer!

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Relativistic Gravitational Force Equation

– *geodesic equation* –

the separation of gravity and inertia: "a mischief of classical mechanics"

$$m_i \frac{dv}{dt} = m_g \frac{MG}{r^2}$$

Newton 2nd Law

Newton Law of Gravity

$$\boxed{U^\nu \nabla_\nu U^\mu = \frac{dU^\mu}{d\tau} + \Gamma^\mu_{\alpha\beta} U^\alpha U^\beta = 0}$$

tensor

<u>"inertia"</u>
not a tensor

<u>"gravity"</u>
not a tensor

$$U^\mu \equiv \frac{dx^\mu}{d\tau}$$

$$c^2 d\tau^2 \equiv g_{\mu\nu} dx^\mu dx^\nu$$

Linear gravity, metric components grouped by rotation properties

$$g_{\mu\nu} \simeq \eta_{\mu\nu} + h_{\mu\nu} \,, \; |h_{\mu\nu}| \ll 1$$

$$h_{tt} \equiv -2\phi \rightarrow scalar$$

$$h_{ti} \equiv w^i = \boldsymbol{w} \rightarrow 3-vector$$

$$h_{ij} \rightarrow 3-tensor$$

Linearized geodesic equation

linear in metric perturbations – linear in test particle speed

$$\boxed{\frac{1}{e}\frac{d\boldsymbol{p}}{dt} \simeq -\nabla\phi - \frac{1}{c}\partial_t \boldsymbol{w} + \boldsymbol{v}\times\nabla\times\boldsymbol{w} - \frac{v^n}{c^2}\partial_t h_{mn}}\ *$$

$$e=\gamma mc^2 \quad , \quad \boldsymbol{p}=\gamma m\boldsymbol{v} \quad , \quad \gamma\simeq\left(1-2\phi+2\boldsymbol{w}\cdot\boldsymbol{v}/c\right)^{-1/2}$$

gravito-electric and gravito-magnetic fields

$$\boldsymbol{E}_g\equiv-\nabla\phi-\partial_t\boldsymbol{w}/c \quad , \quad \boldsymbol{B}_g\equiv\nabla\times\boldsymbol{w}$$

field equations

$$\nabla^2\phi = \frac{\kappa}{2}\rho c^2$$

$$\nabla^2\boldsymbol{w} = -2\kappa\rho c\boldsymbol{U}$$

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*Williams & Inan, Maxwellian mirages in general relativity, arXiv:2012.08077; accepted for pub. New J.Phys.

Einstein's (1921) enumeration of Machian features of GR, based on the truncated linear force equation*

Brans   Sciama   Ciufolini & Wheeler

$$\frac{1}{c^2}\frac{d}{dt}\left[\left(1+\phi\right)\boldsymbol{v}\right]=-\nabla\phi-\frac{1}{c}\partial_t\boldsymbol{w}+\boldsymbol{v}\times\left(\nabla\times\boldsymbol{w}\right)-\cancel{\frac{v^n}{c^2}\partial_t h_{mn}}$$

"1. The inert mass is proportional to 1+Φ, and therefore increases when ponderable masses approach the test body".

"2. There is an inductive action upon accelerated masses, of the same sign, upon the test body".

"3. A material particle, moving perpendicular to the axis of rotation inside a rotating hollow body, is deflected in the sense of the rotation...as shown by Thirring".

*Einstein 1921, Davidson 1957, Brans 1962, Brans 1977, Ciufolini & Wheeler 1995

Brans on inertial induction (1962, 1977): Einstein 1

- “Absence of inertial induction in general relativity”, 1977
- **referred to effect No. 1** from Einstein 1921, an apparent effect on inertial mass from nearby matter, although Einstein used the word “induction” for effect No. 2.
- Never calculated per se, only argued that the Equivalence Principle prevents an external mass from influencing local inertia, because one can always transform to a frame in free fall, where the local mass vanishes.

Sciama (1954) Inertia from gravity (Einstein 2)

weak-field force equation

$$\frac{d\boldsymbol{p}}{dt}=mc^2\left[-\nabla(\phi_U+\phi_L)-\frac{1}{c}\partial_t\boldsymbol{w}+\boldsymbol{v}\times(\nabla\times\boldsymbol{w})\right]\rightarrow set=0$$

1. evaluated in frame of accelerated object
2. total grav force from universe is zero

cosmic grav. potential

$$\phi_U=\int_0^{R_U}G\frac{\rho_U}{r}4\pi r^2dr=2\pi G\rho_U R_U^2=2\pi G\rho_U c^2/H^2\sim c^2$$

$$H^2=\frac{8\pi G}{3}\rho_U$$

Hubble constant H

cosmic vector potential

$$\boldsymbol{w}=\int_0^{R_U}\boldsymbol{v}\frac{\rho_U}{r}4\pi r^2dr=\phi_U\boldsymbol{v}$$

$$-c^2\nabla\phi_L=\partial_t\boldsymbol{v}$$

"Newton Law of Gravity"

Inertia arises from gravitational induction; and if so, then it predicts the mass of the universe

Linear gravity cannot describe inertia

$$h_{\mu\nu} \sim \Phi_U \sim 1$$

- Sciama inertial induction shown only for vector gravity
- Vector gravity only approximates tensor gravity for weak fields
- The gravitational field of the universe is large

Sciama-style inertial induction not proven in GR

C&W on inertial induction: Einstein 3

- Ciufolini & Wheeler (1995) – Gravitation and Inertia – *"inertia here from mass there"*
  - attempted to treat inertia in GR, building on results in MTW. Settled on a gravitomagnetic interaction in the Kerr metric: Einstein 3
  - struggled with boundary conditions. Proposed closed universe with no cosmological constant, yet universe now understood to be 70% CC

$$ds^2 = -\left(1-\frac{2GM}{r}\right)dt^2+\left(1+\frac{2GM}{r}\right)dr^2+r^2\,d\Omega^2-\frac{4J}{r}\sin^2\theta\,d\phi\,dt$$

gravitomagnetism

Does inertia from gravity violate the EP?

~the main conceptual obstacle~

- Sciama showed inertia only for a vector theory of gravity
- Tensor gravity obeys the Equivalence Principle
  - all gravitational fields vanish locally in free fall
- How does a body couple to the gravitational field of the universe if it vanishes in free fall?

Local gravitational phenomena

– Is it possible? –

- If inertia is local, and if it is gravitational, then it is a local gravitational phenomenon
  - such phenomena are unknown to GR, and at odds with the Equiv. Princ.
- If inertia is a local gravitational phenomenon arising from the grav field of the universe, then perhaps momentum can be exchanged locally with the grav field of the universe

Gravitational field of the universe: ~2005

– *flat Robertson-Walker metric* –

$$\boxed{g^U_{\mu\nu}\, dx^\mu\, dx^\nu = c^2 dt^2 - a^2(t)\left[dx^2 + dy^2 + dz^2\right]}$$

$$g_{\mu\nu} = \begin{bmatrix} c^2 & 0 & 0 & 0 \\ 0 & -a^2 & 0 & 0 \\ 0 & 0 & -a^2 & 0 \\ 0 & 0 & 0 & -a^2 \end{bmatrix}$$

$$H^2 \equiv \left(\frac{da/dt}{a}\right)^2 = \frac{8\pi}{3}\rho_U G \equiv t_U^{-2}$$

$$\Phi_U = \int_0^{ct_U} \frac{\rho_U G}{r} 4\pi r^2 dr \propto \rho_U G c^2 t_U^2 \sim c^2$$

Hubble Drag

~force from cosmic metric on a body in motion~

$$U^{\nu}\nabla_{\nu}U^{\mu}=\frac{dU^{\mu}}{d\tau}+\Gamma^{\mu}_{\alpha\beta}U^{\alpha}U^{\beta}=0$$

$$g^{U}_{\mu\nu}dx^{\mu}dx^{\nu}=c^2dt^2-a^2(t)[dx^2+dy^2+dz^2]$$

body with velocity v along x-direction

$$U^{\mu}=(U^t,U^x,0,0)=(U^t,vU^t,0,0)$$

a body in motion experiences drag force from the Hubble expansion

$$\frac{dU^x}{dt}=-2\frac{\dot{a}}{a}U^x=-2HU^x$$

$$H_0t\sim H_0*1000\,s=10^{-14}$$

Force already known from galaxy dynamics.
Also applies to small bodies at a point.

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Lorentz transformation (boosts) of metrics

~gravitational field in moving frame~

$$\tilde{g}_{\mu\nu} = g_{\alpha\beta} \Lambda^{\alpha}_{\mu} \Lambda^{\beta}_{\nu}$$

$$\Lambda^{x}_{t} = \Lambda^{t}_{x} = \gamma \frac{v}{c} \quad , \quad \Lambda^{y}_{t} = \Lambda^{t}_{y} = \Lambda^{z}_{t} = \Lambda^{t}_{z} = 0$$

$$\Lambda^{t}_{t} = \gamma \quad , \quad \Lambda^{x}_{x} = \gamma \quad , \quad \Lambda^{y}_{y} = \Lambda^{z}_{z} = 1$$

$$\Lambda^{i}_{j} = 0 \quad , \quad i \neq j$$

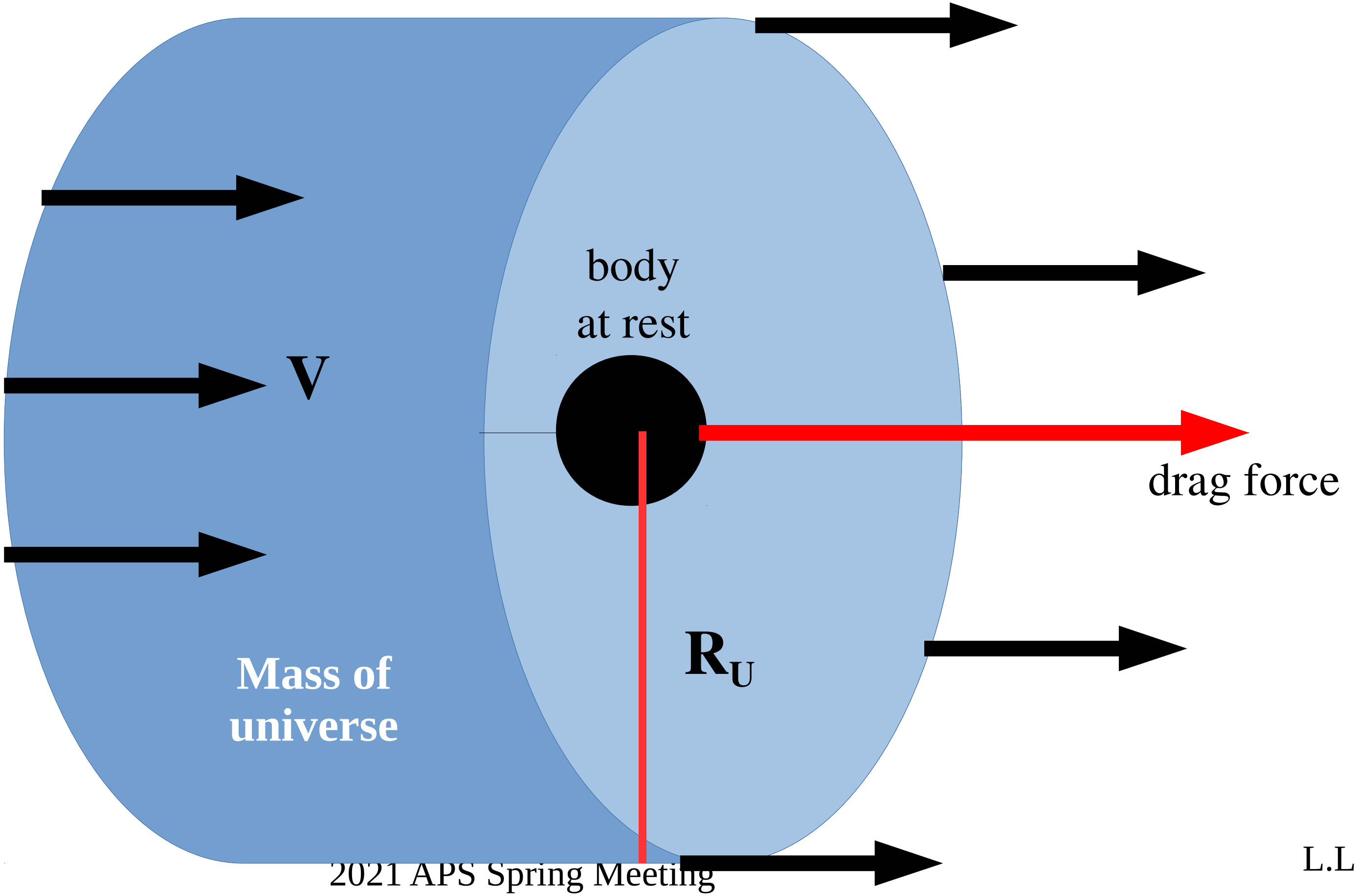
$$\boxed{\tilde{\eta}_{\mu\nu} = \eta_{\alpha\beta} \Lambda^{\alpha}_{\mu} \Lambda^{\beta}_{\nu} = \eta_{\mu\nu}}$$

$$\boxed{g_{tx} = 0 \quad , \quad \tilde{g}_{tx} \neq 0}$$

Gravitomagnetism emerges in the boosted frame
of non-Minkowski metrics

Hubble drag concept from the boosted frame

- A moving object sees a surrounding mass current from the universe
- The motion creates a gravitomagnetic force in the boosted frame
- Confirms Sciama's basic idea
- aka linear frame-dragging



Identification of Rectilinear Frame Dragging

- the Hubble drag effect “looks” gravitomagnetic in all cosmic reference frames except the isotropic frame
- the rest-frame force on moving bodies can be considered frame dragging
- this is the first specific example of rectilinear frame dragging in the literature. Classic frame dragging involves rotating mass fluxes.

Conclusions

- General relativity implies at least 3 non-Newtonian effects on inertia
- They have been conflated in the literature. Brans showed that one was a coordinate effect, but did not consider the other two effects
- The known gravitational field of the universe, the Robertson-Walker metric, induces gravitomagnetic components in boosted frames
- These gravitomagnetic effects lead to a drag force known from galaxy dynamics
- This is the first reported case of linear-geometry frame dragging. Previous work has been for rotational geometries.
- This implies the cosmic gravitational field can be detected in principle locally through inertial-frame experiments with small bodies.