

Optical Behavior of Metals: Radar through Infra-red

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Abstract

This report summarizes the geometric-optics description of the interaction of electromagnetic waves with metals. The components of this description are a complex index of refraction that depends on frequency and conductivity, and the Fresnel equations describing the reflection and transmission of a monochromatic wave incident on a planar surface. From this follows a characterization of the heating of metals at frequencies below the visible, and therefore also of their emissivity over this same frequency range. These results are used to determine heating and equilibrium of a metal surface exposed to earth and sun blackbody radiation fields.

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QUALITATIVE SUMMARY

For sufficiently low radiation frequencies, the optical properties of metals are determined by their conduction electrons. These electrons occupy quantum states that are not bound to any particular atom; rather, they move freely through the ion lattice, carrying thermal energy and electric current. The conduction electrons may be treated mathematically as a gas, more specifically a plasma, contained within the ion lattice. And like a plasma, the ionosphere for example, they respond freely to and reflect radiation of frequencies below the plasma frequency, which for a metal depends only on the number density of conduction electrons. Above the plasma frequency, the metal becomes transparent. For typical metals this occurs in the UV part of the spectrum. Unlike a true plasma, however, the atomic structure of the lattice ions also play a role in the metal optical properties. This behavior, too, manifests at UV frequencies and above. So for frequencies at visible and below, the optical properties of metals can be understood in terms of the conduction electrons.

It is no surprise that metals are reflective in the visible; they are also strongly reflective at lower frequencies as well, through IR and below (assuming the radiation wavelength is still smaller than the physical size of the metal object in question). Although small, absorption does occur at these low frequencies, and is proportional to the square root of the incident frequency.

Now, according to the principle of detailed balance, the emission strength of a metal must equal its absorption strength, as a function of frequency and emission/incidence angle. Therefore, because metals are poor absorbers of radiation, they are also poor emitters. So for two emitting surfaces at the same temperature, a metal will emit *less* than a blackbody. Thus metals will look bright in reflection, cool in emission, compared to blackbodies at the same temperature. And because they are emitting poorly, what radiation they do absorb they hang onto more than a blackbody. They therefore will reach a higher equilibrium temperature in the solar radiation

field than a blackbody. In fact, a metal surface in equilibrium with the solar radiation field will acquire a temperature of around 500 K, quite a bit hotter than a blackbody would acquire in the same radiation field. But metals also have relatively large heat capacities, so it takes them some time to actually reach these temperatures. For example, it takes about an hour to heat a metal slab of thickness 1mm by 100 K in the solar radiation field. The following sections supply the mathematical development that underlies this qualitative description.

INDEX of REFRACTION of METALS

Contribution of Conduction Electrons

Because the index of refraction will form the basis for our considerations of the optical behavior of metals, it is worthwhile summarizing its derivation from scratch. This summary follows Feynman, v1, ch. 32.

As mentioned in the summary, metals are characterized by ‘free’ electrons, which move throughout the lattice of ions comprising the bulk. To a first approximation, these electrons may be treated as a gas, in that they don’t have any bound quantum-mechanical states (the conduction electrons actually move in a band of states in energy space that is so fine-grained it can be viewed as a continuum). When an electromagnetic wave impacts a metal surface, its electric field will shake these conduction electrons, thereby causing *them* to radiate. The entire process can be decomposed into reflected and transmitted waves which characterize the propagation of electromagnetic energy in both materials. If we can come up with an index of refraction for the metal, we can describe this complex process in relatively simple terms. The electromagnetic wave will also interact with the bound electrons, which would require a quantum-mechanical description. Bound states have characteristic energies of an eV, which corresponds to the visible/UV portion of the spectrum. We confine ourselves to the relatively-simple behavior of the conduction electrons. Our description of their behavior will be valid at all frequencies, but the bound electrons will significantly influence the optical properties of metals above visible frequencies.

A conveniently simple and quantum-mechanically correct way of looking at the motion of electrons is to treat them as harmonic oscillators. But since the conduction electrons are free, there is no restoring force. There is, however, dissipation, due to electron collisions with other electrons or with ions. We don’t really know the nature of this dissipation, but let’s say it’s proportional to electron velocity, and introduce a dissipation constant γ . For 1-dimensional motion, with

coordinate x , under the influence of a harmonic electric field E with angular frequency ω , the equation of motion for an electron of mass m and charge q is:

$$qE = qE_0 e^{i\omega t} = m \frac{d^2 x}{dt^2} + m\gamma \frac{dx}{dt}$$

$$\implies x = \frac{qE/m}{i\gamma\omega - \omega^2}$$

This motion induces an instantaneous dipole moment $p = qx$ for each conduction electron relative to its immediate partner ion. Since the atomic polarizability α is defined such that $p = \alpha\epsilon_0 E$ (ϵ_0 is the permittivity of free space, and implies mks units),

$$\alpha = \frac{q^2/m\epsilon_0}{i\gamma\omega - \omega^2}$$

The volume polarization $P = pN = \epsilon_0 N \alpha E$, where N is the number density of these conduction-electron atoms. This last expression is properly written as a vector, in terms of a polarization matrix which represents non-isotropic electromagnetic response of the material. For the conduction electrons, isotropic polarization is a valid approximation.

Since electromagnetic waves are transverse, consider now an electromagnetic wave propagating through our material in the \hat{z} direction, with its electric vector polarized in the \hat{x} direction. The \hat{x} component of the vector wave equation for the electric field (implied by Maxwell's equations) is:

$$\frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = \frac{1}{c^2 \epsilon_0^2} \frac{\partial^2 P}{\partial t^2} - \frac{1}{\epsilon_0} \frac{\partial^2 P}{\partial x^2}$$

The last term is zero, since P follows the spatial variation of E . Our monochromatic electric wave varies as:

$$E = E_0 e^{i(\omega t - kz)} = E_0 e^{i\omega(t - nz/c)}$$

where we have introduced a dielectric constant n such that the wave phase velocity $\omega/k = c/n$; c is the speed of light in vacuum, and k is the magnitude of the wave number.

Substituting these expressions into the wave equation, one finds:

$$n^2 = 1 + N\alpha = 1 + \frac{Nq^2/m\epsilon_0}{i\gamma\omega - \omega^2}$$

What is the meaning of a complex index of refraction? The real part n_R describes the slowing-down of the wave from c to c/n_R . The imaginary part n_I describes damping of the wave with characteristic length $c/\omega n_I$, and will describe energy absorption and therefore electromagnetic heating of the metal, the ultimate goal of these considerations.

The dissipation constant γ can be related to conductivity. But first note that γ is just the inverse of the mean collision time τ for the conduction electrons: Since the collision frequency may be larger or smaller than the wave frequency, a drift velocity v may be defined as simply the product of the instantaneous acceleration qE/m and τ . If we assume that there is no *average* acceleration, that the drift velocity is constant, then we have force balance between drag and the electric force: $\gamma mv = \gamma qE\tau = qE$. Thus $\gamma = 1/\tau$.

For our isotropic metal, a current J will be induced proportional to our electric field, $J = \sigma E$, with σ the conductivity of the metal. But the the average current $J = Nqv$, and therefore the conductivity is related to the collision time/dissipation constant:

$$\sigma = \frac{Nq^2\tau}{m}$$

This allows us to write the index of refraction:

$$n^2 = 1 - \frac{i\sigma/\omega\epsilon_0}{1 + i\omega\tau}$$

Typical Material Parameters

To examine the significance of this expression for n^2 , we must assign some values to the parameters for metals of interest. The table below provides numerical values for the characteristic frequencies of n^2 , based on quantities tabulated in the CRC Handbook of Chemistry and Physics.

	$1/\sigma$ ($\mu\Omega \cdot \text{cm}$)	σ/ϵ_0 (Hz)	$1/\tau$ (Hz)	ω_p (Hz)
carbon steel	18	6.3×10^{17}	4.3×10^{14}	1.6×10^{16}
stainless steel	72	1.6×10^{17}	1.7×10^{15}	1.6×10^{16}
aluminum	3.5	3.2×10^{18}	6.2×10^{13}	1.4×10^{16}
aluminum alloy	7.5	1.5×10^{18}	1.3×10^{14}	1.4×10^{16}
titanium	43	2.6×10^{17}	1.3×10^{15}	1.9×10^{16}
magnesium alloy	9	1.2×10^{18}	2.2×10^{14}	1.6×10^{16}

Calculation of the characteristic frequencies also required mass density, atomic weight, and number of conduction electrons per atom. This latter quantity is in effect a tunable parameter in the expressions, because it follows only from quantum-mechanical considerations of the metal. It was taken to be 2 for titanium and magnesium alloy, 1 for the others. It will generally fall in the range of 1-2.

In the accompanying figures, the index of refraction for a metal is plotted, using representative values of $1/\tau = 3 \times 10^{14}$ Hz, and $\sigma/\epsilon_0 = 5 \times 10^{17}$ Hz. ‘Omega’ on the x-axis refers to the angular frequency of the radiation, which is the product of the frequency in cycles per second and 2π . The imaginary part of n is negative, in keeping with its dissipative character; its absolute value is shown in the plots. The characteristic damping length related to n_I , known as the skin depth, is also shown. In the transition from low to high frequency the imaginary part of n is relatively larger

than the real part, indicating strong absorption at these frequencies. The low- and high-frequency limits of n are considered analytically below.

Low Frequency Limit

It is instructive to examine the low frequency limit of our expression for n^2 . This will be the frequency range where metal optical properties are dominated by the conduction electrons, and will also be of interest for heating in sun and earth blackbody radiation fields. The meaning of 'low' is with respect to the 2 characteristic frequencies: $\omega \ll \sigma/\epsilon_0$ and $\omega \ll 1/\tau$. This implies frequencies infra-red and below. In this case, $n^2 \simeq -i\sigma/\epsilon_0\omega \equiv n_{LF}^2$. Taking the square root:

$$n_{LF} = \sqrt{\frac{\sigma}{2\epsilon_0\omega}}(1 - i)$$

Here, the index of refraction is equal parts real and imaginary. The sign on the square root is chosen such that the sign on the imaginary part implies dissipation. Otherwise, we would have the unphysical situation of the metal feeding the wave amplitudes (which can occur, e.g., in lasing media). We can also easily calculate the low-frequency skin depth δ_{LF} :

$$\delta_{LF} = \sqrt{\frac{2\epsilon_0 c^2}{\sigma\omega}}$$

This value is the exponential damping length of low frequency waves in the metal. It is typically very small. For a nominal value of $\sigma/\epsilon_0 = 10^{18}$ Hz at a UHF frequency of 2.7×10^9 Hz, the skin depth is $8 \mu\text{m}$. Low frequency waves do not penetrate deeply into metals. Their energy dissipation is largely at the surface.

High Frequency Limit

Now consider the opposite limit of radiation frequencies much larger than the characteristic frequencies. Then:

$$n_{HF}^2 = 1 - \left(\frac{\omega_p}{\omega}\right)^2$$

$$\omega_p^2 \equiv \frac{\sigma}{\epsilon_0 \tau} = \frac{Nq^2}{m\epsilon_0}$$

The characteristic frequency ω_p is none other than the plasma frequency that characterizes oscillatory motion of plasmas, and is the frequency at which they become transparent. This transparency sets in when the inertia of the ions is too great to allow them to respond to the radiation. For metals, this frequency is seen to depend only on the number density of the electrons and their mass, not the conductivity. At sufficiently high frequencies, the conduction electrons are effectively a plasma to the radiation, and the metal therefore becomes transparent above its plasma frequency (bound states notwithstanding). Values of ω_p are also tabulated above, and all the metals are seen to have quite similar values. This is because all metals have similar conduction electron densities.

At these frequencies, the dielectric constant is real and the conduction electrons are transparent. This occurs at ultraviolet energies and above. However, we then have to worry about bound electrons at these frequencies leading to absorption bands. By and large, however, metals are transparent in the UV and above. Note that the plots of skin depth show that it can, however, stay below 1mm well into the ultraviolet. A practical definition of transparency, then, is that the thickness of the medium is less than the skin depth.

FRESNEL EQUATIONS

The Fresnel equations relate the amplitudes of reflected and transmitted electromagnetic waves to the amplitudes of the incident waves, when a wave impinges on a planar surface separating two dielectric media. The derivation is algebraically cumbersome, but straightforward. It decomposes the problem into 3 waves: incident, reflected, and transmitted. All are assumed harmonic and monochromatic. Maxwell's equations are used to obtain relations between the electric and magnetic fields on either side of the boundary. It turns out that there are two fundamental types of behavior, depending on whether the incident electric vector is polarized parallel or perpendicular to the plane defined by the incident and reflected wave vectors. The transmitted wave vector is also in this plane.

These equations presume a discontinuous change in dielectric constant, discontinuous meaning the change occurs over a distance much less than the radiation wavelength. Reflection is largely a function of a discontinuous jump in dielectric constant; if the dielectric constant changes sufficiently slowly in space, reflection does not occur. Also, surface irregularities on the scale of a wavelength, or thin layers, can alter the reflection and transmission properties. Although these results assume idealized smooth surfaces and discontinuous boundaries, they provide a good underpinning for understanding how radiation interacts with metals. Because we assume an infinite planar boundary, we effectively assume the physical size of the metal is much larger than a wavelength. This is the regime of geometric optics. When the metal is small relative to the radiation wavelength, or of a similar size, then standing waves can be set up on the target which produce radiation fields not simply describable in terms of simple reflection. In the limit of surfaces small relative to wavelength, a simple description again ensues: scattering cross-section goes as λ^{-4} , a regime known as Rayleigh scattering.

Irrespective of the incident polarization, continuity of the fields at the boundary demands that the reflected and transmitted waves share the frequency of the incident wave: reflection doesn't change frequency. And because $\omega^2/k^2 = c^2/n^2$, k^2/n^2 is a constant among the 3 waves.

The component of the wavevector parallel to the boundary is also a constant among the 3 waves. Measuring the incident angle θ_i with respect to the surface normal, then this component is $k \sin \theta_i = (\omega n/c) \sin \theta_i$. This leads then to Snell's law:

$$\sin \theta_t = \frac{n_1}{n_2} \sin \theta_i$$

where $n_{1,2}$ are the indices of refraction for the (incident,transmitted) waves, and θ_t is the transmission angle. For complex n_2 , this equation implies complex θ_t , and so the transmission angle does not have a simple physical interpretation. The above equation must be considered merely the definition of the quantity θ_t . We now summarize the amplitudes for the two polarizations.

Perpendicular Polarization

In this case, the electric field is perpendicular to the plane of incidence, and is parallel to the boundary surface and unidirectional for all 3 waves.

$$\frac{E_r}{E_i} = \frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t} \quad ; \quad \frac{E_t}{E_i} = \frac{2n_1 \cos \theta_i}{n_1 \cos \theta_i + n_2 \cos \theta_t}$$

where E_i , E_r , E_t are the amplitudes of the incident, reflected, and transmitted electric fields, respectively. The complex transmission angle θ_t is as defined above. That expression may be used to eliminate $n_{1,2}$:

$$\frac{E_r}{E_i} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \quad ; \quad \frac{E_t}{E_i} = \frac{2 \sin \theta_t \cos \theta_i}{\sin(\theta_i + \theta_t)}$$

Parallel Polarization

In this case, the electric field vector is parallel to the incidence plane for all 3 waves. This will be a bit more complicated than the previous case because the incident electric vector now has two components: parallel and normal to the surface. Let us adopt the subscript x for the component normal to the surface, and y for the component parallel to the surface. Then the reflection and transmission coefficients for each of these components are:

$$\frac{E_{rx}}{E_{ix}} = \frac{n_2 \cos \theta_i - n_1 \cos \theta_t}{n_2 \cos \theta_i + n_1 \cos \theta_t} = -\frac{E_{ry}}{E_{iy}} \quad ; \quad \frac{n_2 E_{tx}}{n_1 E_{ix}} = \frac{2n_1 \cos \theta_i}{n_2 \cos \theta_i + n_1 \cos \theta_t} = \frac{E_{ty}}{E_{iy}}$$

In terms of the magnitude of the 3 waves, $E^2 = E_x^2 + E_y^2$, these expressions add up to:

$$\frac{E_r}{E_i} = \frac{n_2 \cos \theta_i - n_1 \cos \theta_t}{n_1 \cos \theta_t + n_2 \cos \theta_i} \quad ; \quad \frac{E_t}{E_i} = \frac{2n_1 \cos \theta_i}{n_1 \cos \theta_t + n_2 \cos \theta_i}$$

Snell's law can be used to eliminate $n_{1,2}$ from these equations:

$$\frac{E_r}{E_i} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} \quad ; \quad \frac{E_t}{E_i} = \frac{2 \sin \theta_t \cos \theta_i}{\sin(\theta_i + \theta_t) \cos(\theta_i - \theta_t)}$$

Normal Incidence

For the special case of radiation incident normal to the surface, the distinction between perpendicular and parallel polarizations vanish.

$$\frac{E_r}{E_i} = \frac{n_1 - n_2}{n_1 + n_2} \quad ; \quad \frac{E_t}{E_i} = \frac{2n_1}{n_1 + n_2}$$

ENERGY ABSORPTION

Reflected Energy

It is a standard result of electrodynamics that the phase velocity of a wave propagating in a material of permittivity ϵ and permeability μ is given by $v = c/\sqrt{\epsilon\mu}$. Restricting ourselves to non-ferromagnetic materials, for which $\mu \simeq \mu_0$, the permeability of free space, then matching this result to our definition of dielectric constant implies $n^2 \propto \epsilon$. Now, the energy density of the electromagnetic field is $\propto \epsilon E^2$, and its flux density in the medium is $\propto (c/n)\epsilon E^2 \propto nE^2$. For a given energy flux density, the quantity entering or leaving a surface is $\propto \cos \theta$, where θ is the angle of the wave vector measured from the surface normal.

Denoting by A , R , T the incident, reflected and transmitted field amplitudes, respectively, then conservation of energy at the surface implies:

$$n_1 \cos \theta_i A^2 = n_1 \cos \theta_r R^2 + n_2 \cos \theta_t T^2$$

or:

$$1 = \frac{R^2}{A^2} + \frac{n_2 \cos \theta_t}{n_1 \cos \theta_i} \frac{T^2}{A^2}$$

where we have used that the angles of incidence and reflection are equal. As long as the skin depth is smaller than the thickness of the metal, requiring only thicknesses greater than 1 mm for radar through visible frequencies, the energy transmitted can be identified as the energy absorbed. Thus the Fresnel reflection coefficients can be used to determine energy absorption:

$$\text{energy absorbed} = 1 - \frac{R^2}{A^2}$$

The intensity of reflected radiation, and its transmitted complement, is shown in subsequent plots as a function of frequency and aspect angle. These plots are obtained by squaring the Fresnel

field amplitude reflection coefficients obtained previously. The incident material was chosen to have $n_1 = 1$, corresponding to vacuum, and n_2 is our canonical metal, plotted in previous figures. Note that fraction of reflected energy goes to 1 as the incidence angle goes to 90° . It is seen that between 10^{16} and 10^{17} Hz, for the choice of parameters, the metal abruptly becomes virtually transparent for all except grazing incidence angles. This is the transition at the plasma frequency. The accompanying color plots show the transmissivity as a function of both incidence angle and frequency.

We also see that a minimum in the parallel energy reflection, and a corresponding maximum in the parallel energy transmission, occurs for grazing incidence angles and frequencies below ω_p . The maximum is more pronounced for frequencies near the characteristic dissipation frequency, $1/\tau$. The same phenomenon occurs for materials with real indices of refraction, and the critical angle is known as the Brewster angle. For real dielectrics, the parallel reflection drops to zero at this angle. The Fresnel equation for parallel waves shows the reflection coefficient $\propto 1/\tan(\theta_i + \theta_t)$. For real transmission angles, this term vanishes as $\theta_i + \theta_t \rightarrow 90^\circ$, when the transmitted and reflected waves propagate at right angles to each other. To understand the Brewster phenomenon in real dielectrics, consider that both the reflected and transmitted waves can be viewed as arising from the same vibrations of surface electrons. Since electromagnetic waves are transverse, and are not radiated along the direction of electron acceleration, the electrons cannot produce two orthogonal waves in the incidence plane, and the parallel reflected wave therefore must drop to zero. For metals, on the other hand, θ_t acquires an imaginary component, and the reflected parallel amplitude therefore remains finite.

Aside from this behavior of the parallel wave, we see from the plots that our canonical metal is greater than 90% reflective over the range of frequencies for incidence angles less than 60 or 70 degrees.

Perfect Absorber

Consider the limit of $n_2 = -in_I$, purely imaginary. Then no propagating transmitted wave exists; the transmitted energy is merely attenuated. Let's further consider the case of normal incidence. Then

$$\left| \frac{E_r}{E_i} \right|^2 = \left| \frac{1 - n_2}{1 + n_2} \right|^2 = \frac{(1 - in_I)(1 + in_I)}{(1 - in_I)(1 + in_I)} = \frac{1 + n_I^2}{1 + n_I^2} = 1$$

For the ideal case of a perfectly absorbing material, all the energy is reflected. A general rule is that if any material gets to be a very good absorber, the waves are strongly reflected at the surface and very little gets inside to be absorbed! We have seen in our plots of the index of refraction for our canonical metal that the imaginary part of the index increases toward lower frequencies. Reflection coefficients get correspondingly large toward these frequencies, with 99% reflection below 10^{14} Hz. Thus, metals are almost perfect absorbers of radiation at infra-red frequencies and below, and are therefore also highly reflective at these frequencies. Although metals are IR reflective, we will see below that because metals are also poor IR emitters, they hold what radiation they do absorb and can be substantially heated in the solar radiation field.

Low Frequency Limit: Absorption and Emission

Although very little low-frequency energy traverses the interface, we can use previous results to quantify what little heating there is in this regime, and thereby thermal emission also. Now consider energy absorption for the low-frequency limit discussed previously, $n \simeq n_{LF}$. Here, $n_R = n_I \gg 1$. For a metal-vacuum interface at normal incidence:

$$\left| \frac{E_r}{E_i} \right|^2 = \left| \frac{1 - n_2}{1 + n_2} \right|^2 \simeq 1 - 2\sqrt{\frac{\epsilon_0\omega}{\sigma}}$$

and thus:

$$\text{fractional energy absorbed} = 2\sqrt{\frac{\epsilon_0\omega}{\sigma}}$$

where we assumed the energy transmitted is absorbed. Thus energy deposition at normal incidence scales as $\sqrt{\omega/\sigma}$ for infrared through radar frequencies. As we saw previously, the fractional energy absorbed is typically less than 1%.

Knowing the absorption of a metal, we can calculate its emission based on standard arguments from thermodynamics. The principle of detailed balance states that energy absorbed at a specific frequency and a specific incidence angle equals the energy emitted at that frequency and angle, for a body in thermal equilibrium at some temperature T . This implies that the ratio of emission to absorption at thermal equilibrium is the same function of frequency and temperature, regardless of material properties. This function is, of course, the Planck blackbody function. Therefore the low-frequency thermal emissivity E_ω of the metal, over a range of frequencies of width $d\omega$, is equal to the product of the Planck function $B(\omega, T)$ and its low-frequency absorptivity:

$$E_\omega d\omega = 2\sqrt{\frac{\epsilon_0\omega}{\sigma}} B(\omega, T) d\omega$$

$$B(\omega, T) \equiv \frac{\hbar}{c^2} \frac{1}{4\pi^2} \frac{\omega^3}{e^{\hbar\omega/kT} - 1}$$

Thus, the low-frequency emissivity of a metal at temperature T is proportional to the Planck spectrum convolved with $\sqrt{\omega/\sigma}$.

An immediate result of the identification of emission and absorption properties is that metals will appear cooler in emission than a less-reflective body at the same temperature. This can be demonstrated in the lab with a tank of water at some temperature. Silver one end of the tank, and blacken the opposite end. Situate thermometers at both ends of the tank. The mirrored end will register a cooler temperature than the blackened end, due to the correspondingly poorer emissivity of the mirror. Thus a metal container of some material at some temperature will emit less radiation than if the container could be treated as a blackbody. When viewed in the infrared,

metal objects will appear cooler, in terms of their emission, than less-reflective bodies at the same temperature. Seen in reflection, metal objects will be infra-red brighter than true blackbodies.

The preceding arguments illustrate the thermal behavior of metals, but the emission formula is not quite right. We actually used the absorptivity for normal incidence. With the exception of the Brewster effect for parallel waves, the reflection is a rather flat function of incidence angle. But as the plots show, there is an incidence-angle dependence which has not been accounted for in the above expressions. Specifically, $B(\omega, T)$ above was obtained by assuming the absorption was not a function of incidence angle. A more-accurate expression for the thermal emission is:

$$E_{\omega} d\omega d\Omega d\alpha = a(\omega, \theta) \frac{\hbar}{c^2} \frac{1}{(2\pi)^3} \frac{\omega^3}{e^{\hbar\omega/kT} - 1} \cos \theta d\omega d\Omega d\alpha$$

This expression gives the power emitted per unit area into a band of frequencies of width $d\omega$, into a unit of solid angle of size $d\Omega$, and of polarization α (there are just 2 polarizations). The angle-dependent absorptivity is $a(\omega, \theta)$. The cosine factor is geometrical, arising from the fact that photon intensity striking or leaving a surface goes as the cosine of the angle measured with respect to the surface normal. The expression for $B(\omega, T)$ was obtained by integrating over Ω , assuming absorption was not a function of angle, and multiplying by 2 to sum over both polarizations. (Note this polarization is different than the polarization of the incident radiation relevant to the Fresnel equations. Photons, being massless particles, have just two polarization states, corresponding to spins parallel or anti-parallel to the direction of propagation.)

Realizing now that there will be a directional dependence to the thermal emissivity of a metal, the previous results using the low-frequency transmission at normal incidence are a suitable representation of the frequency-dependent emissivity for all but oblique angles. The same normal-incidence reflection formulae can also be used with the full expression for the metal index of refraction to obtain a suitable approximation to the metal emissivity valid into the visible.

Heating in Sun and Earth Radiation Fields

The approximation of normal incidence and angle-independent absorption can be used to determine the energy absorption, and therefore the heating, of a metal exposed to the solar or earth blackbody radiation fields.

Let's examine the case of energy absorption by a metal of blackbody radiation at normal incidence, taking specifically the earth blackbody spectrum at 300 K, and the solar spectrum at 5900 K. In the accompanying figure these normalized spectra are superposed on a plot of the normal-incidence transmission factor for our typical metal. Recall that the jump in transmissivity, which we can equate to absorptivity if the skin depth is sufficiently small, occurs at the plasma frequency, which we have seen is relatively insensitive to metal parameters. The 'knee' in the absorptivity at the dissipation frequency is a stronger function of metal properties, and can vary among metals by a factor of 10. Apparently, the UV part of the solar spectrum is falling exponentially right in the frequency range where the metal begins to transmit energy; the earthshine spectrum is negligible in this frequency regime. Although UV can cause appreciable heating in metals while the skin depth remains small, metallic heating in earth and sun radiation fields must be due largely to absorption of a few percent of the incident energy in the peak of its spectrum.

Numerically, the amount of energy absorbed per unit area per unit time will be the integral of the product of the incident spectrum and the frequency-dependent absorption. For normal incidence:

$$\text{energy absorbed} = \int \left(1 - \left| \frac{1 - n_2}{1 + n_2} \right|^2 \right) B(\omega, T) d\omega$$

The solar blackbody spectrum at 5900 K contains about $7 \times 10^7 \text{ W/m}^2$. But this is attenuated by the square of the ratio of the sun's radius, $7 \times 10^8 \text{ m}$, to the earth's distance from the sun, $1.5 \times 10^8 \text{ km}$. The 'solar constant' at earth is thus about 1500 W/m^2 . The earth blackbody spectrum at 300 K contains about 460 W/m^2 , roughly a third of the solar constant. But since the

earth is much closer to objects of interest than is the sun, there is relatively little $1/r^2$ attenuation. For heights h above the earth of radius R_\oplus , where $h \ll R_\oplus$, the distance attenuation factor is about $1 - 2h/R_\oplus$. Heights of 300 km are required to gain a distance attenuation of 10%.

Because of the complexity of the absorptivity dependence on frequency, the above integral must be evaluated numerically. For our metal parameters it is found that roughly 5% of the normally-incident solar energy is absorbed, and roughly 3% of the earth energy. Some insight into the dependence of these factors on material parameters can be gained by considering again the low-frequency limit. Then the above integral simplifies to:

$$\begin{aligned} \text{fraction energy absorbed} &= \frac{\int 2\sqrt{\epsilon_0\omega/\sigma} B(\omega, T) d\omega}{\int B(\omega, T) d\omega} \\ &= 3.8 \sqrt{\frac{kT/\hbar}{\sigma/\epsilon_0}} \end{aligned}$$

Because the earth blackbody spectrum dies exponentially above the assumed dissipation frequency of $1/\tau$, our approximation for the low-frequency absorption is valid for earthshine. Putting $T = 300$ in the above expression yields an absorption of about 3%, consistent with the numerical integral. At low frequencies, the fractional absorbed blackbody energy is proportional to the square root of the ratio of the characteristic thermal frequency to σ/ϵ_0 . Since there can be a factor 10 variation in σ/ϵ_0 among metals, we can expect a factor 3 variation in earthshine absorptivity. Because the magnitude of the index of refraction does not vary much in magnitude between $1/\tau$ and ω_p , the low frequency absorption factor above is approximately correct for solar absorption, even though sunshine peaks in a frequency regime where the metal index of refraction does not have the low-frequency functional form.

So a metallic surface absorbs a few percent of the incident sunshine and earthshine. Is this enough to significantly heat the metal? For a flat metal object of thickness d , density ρ , and specific heat capacity c_p , to raise the temperature by an amount ΔT with a normally-incident

energy flux F requires an amount of time $t = d\rho c_p \Delta T / F$. Some representative metal parameters are tabulated below:

	ρ (g/cm^3)	c_p ($J/g \cdot K$)
Al	2.7	0.9
Cr	7.0	0.45
Co	8.9	0.42
Ag	10.5	0.24

For these values, the time required to raise the temperature of a slab of thickness 1mm by 100 K in the solar radiation field is about 1 hour. For 1cm thickness, 10-15 hours would be required to attain the same temperature increase. Although the earthshine flux is a similar order of magnitude to the solar constant, the earthshine cannot heat the metal beyond the earth blackbody temperature if the metal is in thermal equilibrium with the earthshine environment. For this to occur would violate the 2nd law of thermodynamics. But for a metal object in orbit, or a lower-temperature regime, earthshine heating will proceed as for the solar heating considered above.

Of course, while the metal is absorbing radiation, it is also emitting at a rate governed by its own temperature and emissivity. We have seen that the low absorptivity of metals implies low emissivity, and so a metal object will tend to hold its heat compared to a perfect blackbody at the same temperature. This explains why a coin, key, or other metal object left in the sun can become too hot to touch after a time. How hot could a coin left in the sunshine become? Recall that the integrated energy emitted by a blackbody at temperature T is given by $\sigma_{sb} T^4$, where σ_{sb} is the Stefan-Boltzmann constant (not to be confused with the conductivity σ). We have also seen that the emissivity is equal to the absorptivity. Finally, we have already performed the integral of the Planck spectrum weighted by the low-frequency absorptivity of a metal. Using this low-frequency expression for absorption, and equating the energy received by a metal object to that which it radiates:

$$\left(\frac{R_{\odot}}{A_{\oplus}}\right)^2 \sigma_{sb} T_{\odot}^4 3.8 \sqrt{\frac{kT_{\odot}/\hbar}{\sigma/\epsilon_0}} = \sigma_{sb} T_e^4 3.8 \sqrt{\frac{kT_e/\hbar}{\sigma/\epsilon_0}}$$

$$\implies T_e = T_{\odot} \left(\frac{R_{\odot}}{A_{\oplus}}\right)^{4/9} \simeq 540 \text{ K}$$

In the above, the sun temperature is $T_{\odot} = 5900 \text{ K}$, sun radius is R_{\odot} , earth heliocentric distance is A_{\oplus} , and metal equilibrium temperature is T_e . This expression is strictly valid only for radiation normally-incident on flat plates, and assumes the emitting and absorbing areas are equal. If only one side of the plate was absorbing while both sides were emitting, the equilibrium temperature above would be reduced by a factor $(1/2)^{2/9} \simeq 0.86$. It is interesting that the conductivity cancels out of the expression, implying this equilibrium temperature holds for any metal that obeys an absorptivity proportional to $\sqrt{\omega}$. That is, the functional dependence of the absorptivity, not its magnitude, determines the equilibrium temperature. This is consistent with the idea that thermal equilibrium is independent of material composition. Of course, the amount of time it takes to reach equilibrium is a function of material properties, as considered above. A temperature of 500 K is certainly hot enough to burn organic matter, but is well below the typical melting point of metals, which is about 1000 K.

For a perfect absorber, or a body characterized by an absorptivity independent of frequency, the equilibrium temperature scales like $T_{\odot} (R_{\odot}/A_{\oplus})^{1/2} \simeq 400 \text{ K}$. This is the sort of scaling that determines to first order the ‘equilibrium’ temperatures of the planets in our solar system. That the earth (and the moon) is closer to 300 K than 400 K can be understood in terms of a roughly constant albedo controlling absorption, and perfect blackbody emission (on earth, the clouds, land, and sea play differing roles in emission and absorption, so detailed balance is not violated). In terms of a constant albedo a , this 400 K equilibrium temperature at earth would be reduced by a factor $(1 - a)^{1/4}$; an albedo of 0.7 would bring 400 K down to 300 K.

These same arguments can be used to determine the equilibrium temperature of a metallic plate in orbit, as determined by earthshine. For altitudes of several hundred kilometers, the earthshine equilibrium temperature is:

$$T_e = T_{\oplus} \left(1 - \frac{2h}{R_{\oplus}} \right)^{2/9}$$

At 200 km, the earthshine equilibrium temperature is 295 K, indistinguishable from the equilibrium temperature of the earth itself. Again, the above expression assumes equal emitting and absorbing areas, normal incidence, and low frequencies.

For high altitudes, the scaling is the same as for the solar case. In terms of the geocentric distance of the metal object, R , its equilibrium temperature is given by:

$$T_e = T_{\oplus} \left(\frac{R_{\oplus}}{R} \right)^{4/9}$$

At 5 earth radii, $T_e \simeq 150$ K.

CONCLUSIONS

- Sunshine plays a significant role in the heating of metals, as well as their reflected IR signatures.
- Earthshine does not significantly heat metals, but it will be significant in the reflected IR signature.
- Fraction of IR energy absorbed by a metal $\sim \sqrt{\omega/(\sigma/\epsilon_0)}$.
- Fraction of blackbody energy of temperature T absorbed by a metal $\sim \sqrt{(kT/\hbar)/(\sigma/\epsilon_0)}$
- Metal equilibrium temperatures in the solar radiation field at 1 AU ~ 500 K.
- A region of ‘anomalous’ absorption, and therefore emission, exists for metals at grazing incidence/emission angles or frequencies near the conduction electron dissipation frequency, for radiation with a parallel component of polarization.