The textbook picture of quantum indeterminacy: MIT IAP reference bullet point paper

- The Schroedinger equation describes the evolution of the wavefunction $\psi(x,t)$ of any non-relativistic particle in space and time:

$$i\hbar\frac{\partial\psi}{\partial t} = \left(\frac{-\hbar^2}{2m}\nabla^2 + V(x,t)\right)\psi$$

  There are 4 well-known alternate forms expressing the same mathematical content: Schroedinger wave mechanics (shown here), Heisenberg matrix mechanics, de Broglie/Bohm pilot wave mechanics, and Feynman path integral. Relativistic particles are treated with the Dirac equation; for example, K-shell gold electrons have substantial relativistic mass, but hydrogen electrons do not. Quarks are essentially massless, so the mass of the proton is from the relativistic kinetic energy of its massless constituents (Wilczek, *Lightness of Being*). Schroedinger and Bohm mechanics are not applicable to relativistic particles.

- At this point, the Schroedinger equation is like any other field equation in the laws of physics: it is **deterministic**. It describes the evolution of ψ for all time from specified initial conditions. It's Cauchy problem is well-defined. It is structurally and philosophically akin to the Maxwell equations and the Einstein equations. Quantum wierdness and quantum indeterminacy enter *beyond* the Schroedinger equation.

- **Weirdness #1:** What is ψ? The Schroedinger equation is closely related to the de Broglie relations which preceded it, particle energy = $\hbar\omega$ and particle momentum = $\hbar\mathbf{k}$. Frequency and wavelength of what? de Broglie understood it as a matter wave, like an electromagnetic wave. But that interpretation failed to reconcile theory and experiment. Instead, ψ must be understood as a *probability amplitude* for measuring the position of the particle at any point in space and time. The real probability of any measurement is described by $\psi^2(x,t)$, and $\int \psi^2 dx = 1$. ψ is a *probability wave.* This also establishes the *wave-particle duality* of nature: the state of a **particle** observed in a measurement is governed by a probablity **wave**.

- **Weirdness #2:** Unlike the Maxwell and Einstein equations, the Schroedinger equation doesn't work all the time. It only works when scientists choose not to make measurements. When a scientist makes a measurement, and interacts with a particle to check its "state", she *collapses the wavefunction.* The Schroedinger equation is suspended and ψ is reset, along with its initial conditions and the entire Cauchy problem. This is where consciousness is said to enter the laws of physics, because the scientist chooses to **observe** a particle. This is generally known as *the measurement problem.* (it's not an equation problem!)

- Yet the analogy to Maxwell and Einstein is not complete. In those cases, a test probe does not perturb the system. For quantum scale systems, the measurement by necessity perturbs the particle, and it makes sense our information about the particle, expressed by ψ, would change discontinuously. This sensitivity to perturbation is given by the *Heisenberg uncertainty relations*, $\Delta p \; \Delta x > \hbar/2$, and $\Delta E \; \Delta t > \hbar/2$.

- The philosophical and epistemological issues raised by the 2 weirdnesses have been thoroughly explored and tested. An essential technical, not just philosophical, reference is *Quantum Theory and Measurement*, edited by Wheeler and Zurek (1983). They assemble the most important technical papers informing this question.

- A quantum theory of the electromagnetic field and its interaction with electrons, QED, was subsequently developed from the Feynman picture, and it is the most successful physical theory yet discovered. Constants and measurements are accurate to 13 decimal places. The gravitational constant, by contrast, is known to only 4 places. QED underlies much of our modern telecommunications and microprocessor technology base. Whatever it means, it sure as hell works.

- QED and the quantum weak force are unified in electroweak theory, and the electroweak bosons predicted by that theory are all found since Weinberg wrote down the Lagrangian in 1967. A good quantum theory of the strong force is in place with QCD, which describes the binding of quarks into baryons. It allows us to describe all observed baryons and their resonances seen in accelerator experiments.

- Quantum theory is a tool, like a wrench. It has no separate meaning in and of itself. It is a computer that answers questions. Whatever we think it means, it will work the same. The *Copenhagen interpretation* of the meaning of quantum theory is that nature is fundamentally random at heart, and that we can never predict which nucleus in a pound of uranium will decay next. This is the minimalist interpretation of the laws as we know them, because they contain no more content than this. A *hidden variables* intepretation, or a *many universes* intepretation of quantum theory does not change the power and utility of the tool. It only describes the direction we hope to keep going to find deeper laws of physics. Maybe there are hidden variables. Let us have an X-prize to uncover their laws.

- The fact of wave-particle duality, that neither is a sufficient description of matter, is not necessarily a shortfall of the theory, but rather, a shortfall in our modes of thinking. Wave and particle are like different parts of the elephant we are feeling. If we could conceive of this wave-particle elephant, maybe we would not expect to find meaning in our tools.

- The Feynman path integral expresses ψ as an integration over all possible endpoints, of a phase factor depending on the *action* S between any two endpoints. So $\hbar$ is the *quantum of action.*

  $$\psi(x,t) = \int_{all\ A}^{all\ B} \exp^{iS/\hbar} \; Dx \qquad\qquad S \equiv \int_A^B (mv^2/2 - V) \; dt$$

  The Feynman picture captures best the non-locality, insofar as our puny brains can conceive it, and in relativistic form is the basis of QED.