It's All About Ebno*

*everything you wanted to know about satellite antenna theory but were afraid to ask

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Dramatis Personae

There are 3 important figures of merit at play in RF communications:

- Eb/No: pronounced "ebno". The fundamental figure of merit for digital communications
- G/T : The fundamental figure of merit for receiver antennas
- EIRP: The fundamental figure of merit for transmitter antennas

Introducing Eb/No

The purpose of antenna systems is to receive data encoded in electromagnetic waves, and convert it to standard digital data (1s and 0s) on the ground. If we could get digital data to the ground without electromagnetic (EM) waves, we wouldn't need antennas. For example, the old Corona spy satellites dropped film canisters from space that were scooped up by airplanes.

The quality of the received data is characterized by Eb/No. Eb/No is a digital parameter and it is fundamental to digital communications. It is not specific to antenna systems. It can be intuitively understood as the signal-to-noise ratio for a single bit. Already we can see that for a transmitter of fixed power, increasing the data rate must reduce the energy per bit, and therefore the Eb/No.

Eb denotes energy per bit, and No is the noise level for that bit. The bit error rate of the received data is a function of the Eb/No. Digital communications books show "waterfall curves" that describe the bit error rate as a function of Eb/No. The waterfall curve, and therefore the bit error rate as a function of Eb/No, depends on the details of how the data is modulated on the EM wave.

Introducing EIRP

Now let's consider how energy is transferred between a transmitter and a receiver. These considerations are just basic conservation of energy, and do not assume the data is digital, or that there even is data.

A transmitter emits energy over time. The energy emitted per unit time is the power,. It has units of energy per time (power); watts in MKS units.

The energy flux, energy per area, produced by the transmitter depends on the distance from the transmitter. If the transmitter is emitting radiation equally in all directions (isotropically) with power P_{Ti} , then the flux F_i at a distance R from the transmitter is simply:

$$F_i = \frac{P_{Ti}}{4\pi R^2} \sim \frac{\text{energy}}{\text{time} \cdot \text{area}} \rightarrow \frac{\text{watts}}{\text{meter}^2}$$

As you move further from the transmitter, the power flux drops as the square of the distance. One can capture all the energy from the transmitter by building a small shell around the receiver nearby, or a larger shell around the receiver further away. The far shell will have smaller fluxes (power per area) than the nearby shell, because the same energy is spread over a larger area.

A receiver can only measure the local energy flux. It cannot distinguish whether the flux is due to a weak transmitter nearby, or a strong transmitter far away. Furthermore, it cannot distinguish whether the transmitter is really isotropic.

In fact, most antennas do not emit isotropically. The energy is beamed in a particular direction of interest, by a dish for example. The beamed energy still follows a $1/R^2$ dependence, however. So it is useful to parameterize the transmitter power as if it were isotropic. What the receiver parameterizes as an isotropic power P_{Ti} can actually be decomposed into the actual, non-isotropic power of the transmitter P_T and a transmitter gain G_T . The quantity P_{Ti} is called the "effective isotropic radiated power", EIRP:

EIRP
$$\equiv P_T G_T \sim \text{power} \rightarrow \text{watts}$$

Now we can write the power flux at a distance R, for any transmitter, as

$$F = \frac{EIRP}{4\pi R^2} \rightarrow \frac{\text{watts}}{\text{meter}^2}$$

EIRP is the fundamental figure of merit for a transmitter.

Introducing Received Power Flux

Now that we have parameterized the power emitted by a transmitter in terms of EIRP, consider the power received by an antenna. The received power P_R is just the power flux times the area A of the receiving antenna:

$$P_R = F A = EIRP \frac{A}{4\pi R^2} \sim \text{power} \rightarrow \text{watts}$$

At this point, recourse is made to a mystical result of antenna theory that relates the gain G of an antenna to its area A and the wavelength λ of the radiation:

$$G = 4\pi \frac{A}{\lambda^2}$$

This equation for antenna gain is true for any receive or transmit antenna. It shows how the gain increases as the wavelength of the radiation decreases. Obviously, this expression is wavelength specific, or equivalently, frequency specific. It explains why our dual-band antennas have much higher gain at X-band than at L-band.

Recall that electromagnetic waves of all frequencies travel at the speed of light c, and the relation of frequency ν to wavelength is

$$c = \lambda v$$

In the expression for gain, the effective area of the antenna is not necessarily the physical area of the antenna dish. The parameterization is called the *antenna efficiency*. Typical values of antenna efficiencies range between 0.5 and 0.7. A nominal value for satellite systems is 0.55. This means the effective area of an antenna dish is about half the physical area. Another way to say it is that a typical antenna is able to collect only about half the energy that strikes it.

Now we use the expression for receiver gain to substitute for receiver effective area in the expression for received power:

$$P_R = (EIRP)G_R \left(\frac{\lambda}{4\pi R}\right)^2 \sim \text{power} \rightarrow \text{watts}$$

The quantity in parentheses on the right is called the *free space path loss* in link margin calculations. It's an oddball quantity, as a ratio of distance to wavelength, but it allows us to speak in terms of antenna gain instead of antenna effective area.

Introducing Noise Temperature

The "T" in G/T is antenna noise temperature. Noise temperature is used to parameterize the noise energy in an antenna system. Noise energy is the energy an antenna receives in the absence of any signal, just pointing to cold space or empty sky.

To parameterize in terms of a noise temperature requires the use of the Boltzmann constant, k. Boltzmann is a fundamental number in nature. Even though it was discovered through thermodynamics, it appears in quantum theory. It provides the measure of the amount of energy contained in thermal motion. A body of temperature T is understood to be composed of microparticles, each having a kinetic energy $\sim kT$.

Miraculously, Boltzmann also allows us to parameterize noise energy for a system in thermodynamic equilibrium (the temperature isn't changing). Since kT has units of energy:

$$k \sim \frac{\text{energy}}{\text{temperature}} \sim \frac{\text{power}}{\text{temperature}/\text{time}} \sim \frac{\text{power}}{\text{temperature} \cdot \text{frequency}}$$

Although it has not been proven, just shown in dimensional analysis here, the Boltzmann constant tells us how much noise power exists per unit temperature per unit frequency. Therefore kT describes the noise power spectrum, the amount of noise power in an increment of frequency.

The *noise power spectrum* is therefore

$$N_0 \equiv kT \sim \frac{\text{power}}{\text{frequency}} \rightarrow \frac{\text{watts}}{\text{Hz}}$$

White noise by definition has constant noise power (constant temperature) across all frequencies.

There is also a commonly-used quantity that describes the *noise power* contained within a frequency bandwidth Δv :

$$N \equiv kT\Delta\nu \sim \text{power} \rightarrow \text{watts}$$

The common notation for noise power is N, and N_0 is used for noise power spectrum.

Keep in mind that the noise temperature over a frequency of interest may not necessarily be the physical temperature of the system. And the noise temperature can be different at different frequencies. The noise temperature is a parameterization of noise that may have no connection to the physical temperature of the system.

Introducing Carrier-to-noise

Carrier-to-noise is a ratio that is calculated intermediate to calculation of Eb/No. Carrier-to-noise (ratio) is obtained by forming a ratio of the received power P_R to the noise power spectrum N_0 :

$$C/N_0 \equiv \frac{P_R}{kT} = \frac{(EIRP)G_R}{kT} \left(\frac{\lambda}{4\pi R}\right)^2 \sim \text{frequency} \rightarrow \text{Hz}$$

This is basically a signal-to-noise ratio, but it has units of frequency. It is always written as C/N_0 , and sometimes it is called "c-over-n-zero". There has been no assumption about the nature of the radiation or what it is carrying, so this equation is quite general for electromagnetic systems. If the ratio is taken to N instead of N_0 , a true signal-to-noise ratio is obtained, and we have something very similar to the *radar equation*.

Now to Eb/No

At this point, we investigate the case where the electromagnetic wave is carrying digital data. The digital data transferred is described by its data rate, D. The units of data rate are bits per second. Bits are unitless, so the units of data rate are inverse time, equivalent to frequency. I will not show why, but you can see from the units that if we ratio the carrier-to-noise to the data rate, we get a true unitless signal-to-noise ratio. This ratio is Eb/No:

$$E_b/N_0 \equiv \frac{C/N_0}{D} = \frac{P_R}{kTD} = \frac{(EIRP)G_R}{DkT} \left(\frac{\lambda}{4\pi R}\right)^2$$

The EIRP can be anything, and the data rate can be anything. A fixed EIRP at a higher data rate will have less energy per bit. Therefore, Eb/No is a figure of merit for digital data that intuitively indicates the signal-to-noise ratio of each bit, where a bit is defined by its period, the inverse of the data rate. This is the fundamental figure of merit for the transfer of digital data. Given a choice of modulation, the output bit error rate will be a function only of Eb/No (sometimes called "E-B-over-N-zero").

Introducing G/T

In the equation for Eb/No, the EIRP depends only on the transmitter. It can be constructed from a higher gain antenna with a lower power amplifier, or a lower gain antenna with a higher power amplifier. The Eb/No can't tell the difference. Hence only the product of power and gain is relevant to the Eb/No. Antenna designers will spec the transmitter gain and the amplifier separately, and let data weenies construct the EIRP. Conversely, data weenies will spec only the EIRP, and the transmitter designer is left to trade power and gain.

Likewise for the receiver: the quantities in the Eb/No equation that depend on the receiver are the receiver gain and receiver noise temperature. They enter as a ratio, and the Eb/No can't distinguish a higher gain receiver with higher noise, and a lower gain receiver with lower noise. Therefore the receiver parameters G_R and T are combined in a single figure of merit, G/T. Again, antenna designers will spec both separately, but a data weenie will spec G/T, and let the antenna designer trade noise and gain. So we can rewrite the Eb/No expression, grouping terms:

$$E_b/N_0 = (EIRP) \left(\frac{G}{T}\right) \frac{1}{Dk} \left(\frac{\lambda}{4\pi R}\right)^2$$

This expression provides the data figure of merit in terms of the transmitter figure of merit, the receiver figure of merit, the data rate, the free space loss, and the Boltzmann constant. This is the standard *link margin equation* for digital RF communications. Generally, the Eb/No should be 10 or greater for good comm.

I will note here that use of *forward error correction* provides a way to get high data quality at lower Eb/No, in exchange for increasing bandwidth. That is another story that is beyond this tutorial. Be advised that forward error correction is widely employed on space links. Some common examples of forward error correction are convolutional encoding (the decoding is called Viterbi), Reed-Solomon, and LDPC. However, an antenna guy need not worry about this, because the forward error correction looks just like the rest of the "data." So we are justified in ignoring it here.

G/T Calibration with Known Radio Sources

Some satellite antennas are calibrated with CasA, a radio source in the constellation Casseopeia, which we now know to be a giant black hole at the center of the galaxy. Its value is because the radio spectrum (power received per unit frequency) is well characterized. A known power spectrum Φ from a known source can be used to calibrate an antenna. The reference source has units

$$\Phi(\nu) \sim \frac{\text{power}}{\text{area} \cdot \text{frequency}} \rightarrow \frac{\text{watts}}{\text{meter}^2 \cdot \text{Hz}}$$

When an antenna is not pointed at any transmitter, it still receives noise energy. The antenna and its entire environment are underwater in a giant swimming pool of noise energy, and it is the same in any direction (ok, it can depend on direction but ignore that for now).

When an antenna is pointed at a transmitter, it receives the energy from the transmitter plus the noise energy, because you can never escape the noise energy. Therefore, when we measure the power from the radio source P_S , we are also receiving power P_N . Therefore a source measurement $M_S = P_S + P_N$, and a noise measurement $M_N = P_N$. A G/T measurement is obtained by differencing these two and ratioing them to the noise measurement (refer back to the results on received power flux)

$$\frac{M_S - M_N}{M_N} = \frac{P_S}{P_N} = \frac{\Phi A \,\Delta\nu}{k \,T \,\Delta\nu} = \frac{\Phi}{kT} \frac{G\lambda^2}{4\pi} = \Phi \frac{G}{T} \frac{\lambda^2}{4\pi k}$$

where we have expressed the received power in terms of the bandwidth Δv of the receiver, which is the same whether measuring the source or the noise only. We have also used the mystical result which relates antenna gain to antenna area. Invert this equation to obtain

$$G/T = \left(\frac{M_S}{M_N} - 1\right) \left(\frac{4\pi k}{\Phi \lambda^2}\right)$$

The right hand side is in terms of known quantities and the two measurements M_S and M_N . There are correction factors applied to this basic formula, but this illustrates the basic concept. The measurement term in the expression is sometimes called the *Y*-factor. The quantity Φ is read off a lookup table for CasA, evaluated at the frequency at which the measurement is taken.

Introducing Decibels

The parameters in the link margin equation involve some very large and very small numbers. To handle these, RF communications are typically described in terms of *decibels*, abbreviated dB. A decibel is a logarithm.

$$dB(x) \rightarrow 10 \log x$$

The logarithm is base 10, so 100 = 20 dB, 1000 = 30 dB, and so on.

This allows us to write the link margin equation as a sum instead of a product:

$$dB(Eb/No) = dB(EIRP) + dB(G/T) - dB(D) - dB(k) + dB(FSL)$$

where the free space loss is written FSL.

A useful rule of thumb is that 3 dB is approximately a factor of 2. Then 6 dB is a factor 4, 9 dB is a factor of 8, and so on.

Transmitter EIRP is typically measured in milliwatts, and decibel-milliwatts is written dBm. Sometimes you see watts, and then a decibel-watt is written dBW, = 30 dBm.

Noise temperature is typically measured in kelvin, and decibel-kelvin is written dB-K. Since gain is unitless, G/T is measured in decibel-inverse-kelvin, dB/K.

Introducing Modulation

Now that we have discussed how the power in an electromagnetic wave translates to digital data signal-to-noise, let us consider how data is *modulated* on the wave.

Start by considering a simple sine wave. Such a wave has 3 independent parameters: amplitude *A*, frequency ω (in radians per second), and phase ϕ (in radians):

$$W(t) = A\sin(\omega t + \phi)$$

Here, t is a time coordinate. This describes the function W(t) at a fixed position in space.

If we want to encode data on the wave, we can encode the amplitude, the frequency, or the phase.

Amplitude modulation at constant phase and frequency looks like this:

$$W(t) = A(t)\sin(\omega_0 t + \phi_0)$$

This is how data is encoded on AM radio. You tune the radio to a fixed frequency, and the information is in the time-dependent amplitude. In the old days, AM radio was analog, and A(t) was a smoothly varying function.

To modulate digital data, you would have two values of amplitude, and switch back and forth between them each bit period. Amplitude modulation of digital data is called *amplitude shift keying*, ASK. The amplitudes change abruptly and discontinuously with the data bits.

Frequency modulation at constant amplitude and phase looks like this:

$$W(t) = A_0 \sin(\omega[t] t + \phi_0)$$

This is how data is encoded on FM radio. In this analog case, the frequency you tune the radio to is a center frequency about which the frequency goes up and down, according to the data. This means FM radio channels have to have enough frequency separation between them to contain the modulated data without impacting the adjacent FM channel.

Frequency modulation of digital data is called *frequency shift keying*, FSK. The frequencies change abruptly and discontinuously with the data bits. The old Air Force satellite links used a 3-tone FSK scheme. But FSK is frowned upon nowadays because it uses a lot of bandwidth, and spectrum is precious. However, FSK can be useful in certain situations, and there are a wealth of FSK waveforms.

Phase modulation at constant frequency and amplitude looks like this:

$$W(t) = A_0 \sin(\omega_0 t + \phi[t])$$

Here the data is encoded in a phase change. You should look up some pictures of this and the other modulation types to compare. A phase change at constant frequency looks kind of funny. I am not aware of a well-known example of analog phase modulation.

Phase modulation of digital data is called *phase shift keying*, PSK. The phases change abruptly and discontinuously with the data bits. This modulation type is dominant in communications today because it is bandwidth-efficient.

If there are two phase values, say 0 and 180, then this type of modulation is called *binary phase shift keying*, BPSK. This is perhaps the dominant mode of encoding digital data today. It is simple and robust. Each phase value corresponds to a 1 or a 0, and the modulated phases are changing at the data rate.

If there are four phase values, say 0, 90, 180, and 270, then this type of modulation is called *quadrature phase shift keying*, QPSK. This is also a very widespread modulation technique today. It is closely related to BPSK, and a QPSK signal is often decomposed into two BPSK signals. These two BPSK channels in a QPSK waveform are called "I" and "Q". QPSK and BPSK have the same bit error rate as a function of Eb/No.

The beauty of QPSK is that, with four phase values, each phase value can encode two bits: 00, 01, 10, 11. This means that the modulated phases are changing at one half the data rate. Put another way, the output data rate is twice the rate of the input phase variation.

We can keep going: *8PSK* uses 8 phase values: say, 0, 45, 90, 135, 180, 225, 270, 315. Now each phase value corresponds to 3 bits: 000, 001, 010, 100, 011, 101, 110, 111. This means that the 8PSK waveform is waving at 1/3 the rate of the underlying data.

Wait. Do I smell a free lunch? Can we keep going and subdivide the phase circle into a bunch of narrow slices, thereby sending a huge data rate at a very low modulated phase rate? The answer is no. I refer you at this point to the *Shannon limit*, and other results developed by Claude Shannon. They are considered some of the most beautiful results in mathematics and engineering.

The problem is that the detector does not see sharp phase values. Even a BPSK detector may see values of 90 or 270, in which case the detector has a 50-50 chance of assigning the wrong phase value. QPSK has 4 of these boundaries, and a higher probability of error. Higher rate phase modulation types get into more trouble at the detector. So you can stuff more data bits into a narrower phase slice, but you get higher errors.

Shannon derived the minimum Eb/No to send error-free data, without telling you exactly how it is encoded. He said it is out there, go look for it. The science of modulation and forward error correction is one of striving to send data at the Shannon limit. (BTW, thanks to modern microprocessors, we now have techniques that operate near the Shannon limit. DVB-S2 is just about there).

Introducing Polarization

For electromagnetic waves, the thing that is waving is the electric field. There is an associated magnetic field that is also waving, but the magnetic part is typically ignored because the electrons in your antenna are moved by the electric field, not the magnetic field. Electric fields are what moves electrons in metals, whether antennas or power lines..

The electromagnetic waves that are carrying our digital data have an important property: they are *transverse waves*. That means that the electric field waves in a direction transverse, or perpendicular, to the propagation vector of the wave itself.

For a simple wire antenna to receive electromagnetic radiation, the axis of the antenna must be parallel to the waving electric field. If the antenna axis is perpendicular to the waving electric field, the electrons cannot respond. Because the wave is transverse, the axis of the antenna should be perpendicular to the advancing electromagnetic wave. If the antenna axis lies along the direction of wave propagation, then the waving electric field is perpendicular to the antenna, and the antenna electrons cannot respond to the waving electric field.

When the propagation vector of the radiation is specified, the electric field vector is confined to the two-dimensional plane perpendicular to the propagation vector. If this plane is represented as an X-Y diagram, with the propagation along the Z-direction, then the electric field vector can point anywhere in the plane. The direction of the waving electric field in such a plane is called the *polarization* of the electromagnetic wave.

In general, unless you are looking at a laser, the radiation will consist of polarizations in all directions in the plane perpendicular to the propagation vector. The same is true for the sun or a light bulb. Projected on the plane, there will be energy with all polarizations from 0 to 360 referenced to this plane. Polarized sunglasses reduce glare by screening out all those polarization directions except one.

A neat result of the transverse nature of light is that the light from a blue sky is polarized when viewed at 90 degrees from the sun. Draw yourself a picture to see why this is so, if it is not immediately obvious. It is easy to test with polarized sunglasses: look at the sky 90 degrees from the sun, and rotate the lens until the blue sky goes dark. This will be true for any radiation scattering in a medium, when viewing the medium perpendicular to the radiation propagation vector.

It is conventional for ground antenna systems to set up a coordinate system for this X-Y plane containing the electric field vector (the polarization) such that the component vertical with respect to the ground (surface of the earth at the antenna) is called "V", and the component horizontal with respect to the ground is called "H". This choice is arbitrary, but the electric field vector is real. Because the polarization can be anywhere in the plane perpendicular to the propagation direction, two components of polarization are necessary to specify the wave.

Transmitters typically emit polarized radiation, so one has to make sure the receiver polarization is aligned with the transmitter, or else energy will be lost to the receiver. To avoid this, satellite communications typically use *circularly polarized* radiation. This is obtained by phase shifting the two linear polarization components (X and Y, or H and V) by 90 degrees. Then the polarization vector rotates as the wave travels. When viewing oncoming circularly polarized radiation, if the electric field vector rotates counterclockwise, it is called right hand circularly polarized (RHCP); if clockwise, then left hand circularly polarized (LHCP).

So circularly polarized radiation always has V and H components (linear polarization) that change in time, and a linear polarized receiver will always detect a fraction of the radiation. Use of circular polarization avoids the problem of aligning the polarization of the transmitter and receiver. One can always express linear polarization in terms of the two RHCP and LHCP components; conversely, one can always express circular polarization in terms of the two linear components, H and V.

Introducing Bandwidth, Bandpass, Passband, Baseband, Boyband

Bandwidth refers to the amount of spectrum (frequency domain) that is occupied by data at a given rate. *The bandwidth is approximately the data rate in hertz*. An S-band carrier may be at 2 GHz frequency, but the bandwidth of the signal depends on the data rate of the encoded data it carries. So bandwidth is the amount of frequency centered on the carrier that is occupied by the data.

The frequency of the carrier must be larger than the bandwidth of the data. Obviously, you cannot put 10 MHz of data on a wave of frequency below 10 MHz. In general, higher rate data requires higher rate carrier frequencies. A rule of thumb is that the carrier frequency will be 100 to 1000 times the data rate or more. That is why relatively low frequency S-band carries relatively low rate data, and higher frequency X-band carriers higher rate data. Modern satcom now uses Ka-band, at around 30 GHz, to carry data rates up to around 300 Mbps.

Baseband refers to the frequency domain of just the data itself, before it is modulated on a carrier. So the bandwidth "at baseband" is approximately the data rate in hertz.

Passband refers to the frequency domain after the data has been modulated on the carrier. Due to the physics of modulation, the bandwidth of the data "at passband" is double the bandwidth at baseband. This term is used in the context of filtering, where some parts of the band do not "pass", and other parts do: the passband.

Bandpass is about the same as passband. Bandpass just refers to the frequency domain after the data is modulated on the carrier ("at bandpass").

Boyband is the modulation of hormonal data onto high-pitch acoustic noise. Boyband hormonalacoustic communication has nothing to do with satellite communications or electromagnetic radiation, but I mention it here for completeness.

Introducing Intermediate Frequency (IF)

It is impractical to demodulate data directly from S-band or X-band frequencies. Any system that tried to do so would generate large, high-frequency electric fields, and your demodulators would spark and sizzle like Tesla's laboratory. Furthermore, if we attempted to demodulate directly from passband to baseband, we would need to tune a separate oscillator in the modem for each carrier frequency supported by the ground system.

Therefore it is convenient to downconvert all received radiation to the same relatively lowfrequency "intermediate frequency." Then the demodulators can work on data received at different passband frequencies, by tuning the demodulator to a single intermediate frequency. And also, the demodulators can perform better at relatively low frequencies. A standard IF for Sband and L-band is 70 MHz. Standard IFs for X-band are 720 MHz and 1.2 GHz.

Again, the IF must be high enough to contain the modulated data. You cannot downconvert Kaband data at 300 Mbps to 70 MHz – it's physically impossible. That is why bands of increasing frequency require IFs of increasing frequency – because bands of increasing frequency are carrying data of higher bandwidths.

Introducing Antenna Beam Size

Antennas receive and transmit in space according to a lobed geometry. The main lobe is on the antenna dish centerline, but there are also sidelobes all the way out to 90 degrees. They are the diffraction pattern of the antenna. The sidelobes are always smaller than the main lobe, meaning the antenna is not as sensitive in those directions as it is at *beam center*.

The width of the main lobe, through which most of the energy is received or transmitted, is called the *beamwidth*. There are several ways of defining beamwidth, such as distance between the adjacent nulls bracketing the main lobe, or the distance between the 1/2-power (3 dB) points. If the target for reception or transmission moves beyond the 3 dB points, the signal begins to drop. On the other hand, it is pointless (ha ha) to try to point the antenna with an angular resolution less than a tenth of the beamwidth or so, and a strong signal can be received even if the source and receiver are not at beam center, as long as the deviation is less than a beamwidth.

A useful simple formula for 3 dB beamwidth θ is

$$\theta = \lambda/L \rightarrow \text{radians}$$

where λ is the wavelength of the radiation and *L* is the diameter of the dish.

For S-band at 2 GHz on the 16-meter, $\theta = c/(2 Ghz)(16) = 0.6$ degrees. The beam width for X-band at 10 GHz with the same dish is 0.1 deg. X-band signals will have a tighter tracking requirement than S-band signals because of the smaller beamwidth.