# Acceleration in the frame of the wave: hydrodynamic lift in surfing

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**Abstract**
GPS technology has recently been used to measure the speed of a surfer on a wave. The measured speeds are upwards of 80 km/hr. The two main contributions to a surfer's speed found in conventional analyses of the physics of surfing, gravitational acceleration from sliding down the wave, and the speed of the wave itself, do not appear to account for the high measured speeds. A surfer that drops down a wave can be seen shooting back up well over the top of the wave, obviously with more energy than could have been gained by dropping into the wave. There appears to be a source of thrust moving in the frame of the wave. This article suggests the thrust is due to hydrodynamic lift. Furthermore, the nature of the lift effect is so generic that it is relatively insensitive to the shape of the surfboard; it can be experienced surfing on a plank.

## I. Introduction

The basics of surfing is to catch a wave – to accelerate into the rest frame of the wave and ride the wave into the beach. The art of surfing is what happens after the wave is caught. Basic explanations of surfing tend to focus on the physics of catching the wave. It appears to not be widely recognized that the aerial gymnastics of a surfer are achieved by tapping into a source of thrust moving in the rest frame of the wave. With the advent of GPS-based measurements of surfer motion, it has become clear that the speed of the wave plus the speed of falling down the wave face do not provide all the kinetic energy of a surfer. Surfers are seen to have speeds with respect to the beach of greater than 80 km/hr. [1]

Even an unconscious surfer can catch a wave, if given an opportune push. But a skilled surfer apparently taps into a limitless reservoir of energy. Here we argue that the extra energy comes from hydrodynamic lift – the turning of a flow.

This article starts with a review of the speed and height of surf waves, and compares the energy budget from wave motion and falling down the wave with the observed GPS speeds of some surfers. We then describe how hydrodynamic lift of the surfboard can provide extra thrust to propel a surfer to high speeds in the frame of motion of the wave.

## II. Estimating surf wave speeds

Ocean waves are influenced by their own momentum, by gravity, by water pressure, by viscosity, and by surface tension. The equation which describes these effects (except surface tension) is easy to write down:

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} + \rho(\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = \rho \boldsymbol{g} - \nabla P + \eta \nabla^2 \boldsymbol{v} \quad . \tag{1}$$

Equation (1) is the famous Navier-Stokes equation of fluid dynamics. It is the equation describing the physics of ocean waves. It can be written down, but not so readily solved. The mathematical challenge is that equation (1) is non-linear; there are no general simple solutions as there are in other areas of physics, such as planetary motion. The non-linearities which confound simple mathematical solutions arise from the momentum effects alone, the second term on the left hand side of (1), not due to complications introduced by the other effects. Techniques exist to obtain some analytical solutions which still describe much of what a surfer may experience with ocean waves. To do much better than the analytical estimates which follow requires a full-up numerical simulation. This article will ignore dissipation from viscosity, surface tension, or churning of the sea bed, and use idealized results to bound the phenomena.

For surface waves propagating over a three-dimensional fluid of arbitrary depth, one obtains some useful simplifications of equation (1) by assuming the flow is irrotational, incompressible, and ignoring viscosity. One further ignores the non-linear terms, corresponding to assuming the wave heights are much less than the wavelengths. For one-dimensional surface waves of this sort, there are two different wave speeds depending on whether the waves are in deep water or shallow water.

A deep water wave travels over a depth, *d*, much greater than its wavelength, $\lambda$: $d > \lambda$ . The speed $V_{deep}$ of such a wave depends on its wavelength and on the acceleration of gravity, *g*: [2]

$$V_{deep} = \sqrt{\frac{g\lambda}{2\pi}} = \frac{g\tau}{2\pi} \quad . \tag{2}$$

The last equality follows since $V_{deep} \equiv \lambda / \tau$ , where $\tau$ is the wave period. Selections from (2) for a parameter range of interest are shown in Table I.

**Table I. Deep water wave characteristics**

| Wave period (seconds) | Wavelength (meters) | Deep water speed (km/hr) |
|---|---|---|
| 1 | 1.6 | 6 |
| 2 | 6.3 | 11 |
| 3 | 14 | 17 |
| 5 | 40 | 30 |
| 10 | 160 | 60 |
| 15 | 360 | 90 |
| 20 | 640 | 120 |
| 25 | 1000 | 150 |
| 30 | 1400 | 180 |

Energy and momentum, however, are not necessarily delivered at the deep water wave speed. Rather, it is delivered at the group speed, which works out to be one half the deep water speed:[3]

$$V_{group} = \frac{V_{deep}}{2} \quad . \tag{3}$$

As a large wave moves in to shore, one may see a retinue of smaller waves running backward and forward through the large pulse of energy-delivering waves, with the large pulse moving at half the speed of the retinue waves. It is likely these larger, half speed, energy-carrying waves will be the ones most attractive to a surfer. [4]

A shallow water wave travels over a depth, *d*, much smaller than its wavelength, $\lambda$: $d \ll \lambda$ . The speed $V_{shallow}$ of such a wave depends only on *d* and *g* for waves of any length or period:[5]

$$V_{shallow} = \sqrt{gd} \quad . \tag{4}$$

Selections from (4) for a parameter range of interest are shown in Table II.

**Table II. Shallow water wave characteristics**

| depth (meters) | Shallow water speed (km/hr) |
|---|---|
| 0.3 | 6 |
| 1 | 11 |
| 3 | 20 |
| 10 | 36 |
| 30 | 62 |
| 100 | 110 |

These shallow wave speeds are generally much less than the deep water speeds, so the waves pile up as they reach the shore. Furthermore, the wave speed at the crest will be higher than at the trough because the crest is deeper than the trough, and the crest will overtake the trough. This is how the waves begin to break.

## III. The energy budget of gravity and wave motion

Returning to the topic of achieving 80 km/hr on a wave, Table II shows that the long period deep water waves move at these speeds. Consider the 15 second wave. It has a wavelength of a third of a kilometer, and it will start to shoal in water 1/4 that deep[6], 90 meters. The shallow wave speed for that depth is upwards of 100 km/hr. Waves start to break when the breaking depth $d_B \sim 1.3\, H_B$ , where $H_B$ is the breaker wave height.[7] A break at 40 meters depth would imply a wave height of 30 meters.

The high shallow water speeds for big waves explains the need for tow-in to surf them. Yet it seems that breaking, surfable waves can occur in as little as 3 meters of depth, with corresponding shallow wave speeds of 20 km/hr or less and breaker heights of several meters. Just the fact that the surfer was able to catch the wave implies its speed was no more than perhaps 10 km/hr, and it must slow down as it reaches the shore according to the shallow wave speed. Other treatments of the physics of surfing (e.g., Ref. 4) may focus on how the wave accelerates the surfer up to wave speed, but for our purposes it is enough to know that the surfer eventually ends up moving on and with the wave. Even so, the wave alone is not sufficient to explain 80 km/hr.

There is another source of energy exploited by the surfer: gravity. If the surfer catches the wave as it is breaking, he can tip over and slide down the face of the breaking wave. As the wave grows or rolls long enough for a fall down the face of the wave, the surfer will gain potential energy from gravity and turn it into kinetic energy at the bottom of the wave.

An object falling from a height *H* would gain a speed

$$V_{grav} = \sqrt{2gH} \quad . \qquad (5)$$

Selections from (5) for a parameter range of interest are shown in Table III.

**Table III. Surf wave fall speeds**

| Fall Height (meters) | Fall Speed (km/hr) |
|---|---|
| 0.3 | 9 |
| 1 | 16 |
| 3 | 28 |
| 10 | 51 |

Now let's combine the energy available from the wave speed plus the energy from free fall. Consider waves breaking in depth $d_B \sim 1.3\, H_B$ . Then the wave speed and the breaker height are related by $V_{wave} = \sqrt{1.3\, g\, H_B}$ . Since the surfer moves always parallel to the wave face, then the total speed due to gravity and wave motion, in terms of the wave height *H,* is obtained from:

$$V_{total} = \sqrt{V_{grav}^2 + V_{wave}^2} = \sqrt{3.3\, gH} \quad . \qquad (6)$$

Selections from (6) for a parameter range of interest are shown in Table IV.

**Table IV. Surfer speeds available from wave motion and falling down the face**

| Wave Height (meters) | Wave speed (km/hr) | Wave speed/height (mph/ft) | Fall speed (km/hr) | Total speed (km/hr) |
|---|---|---|---|---|
| 0.3 | 7 | 4.4 | 9 | 11 |
| 1 | 13 | 2.5 | 16 | 21 |
| 3 | 22 | 1.4 | 28 | 34 |
| 10 | 41 | 0.8 | 51 | 65 |

It is apparent from the generous estimates above that the main parts of the energy budget, gravity plus wave speed, do not account for high surfer speeds on typical-size waves. Table IV also explains the surfer's rule of thumb of 1 mile per hour of speed per foot of wave height.

If the surfer had only the energy of gravity to work with in the frame of motion moving with the wave, then he would just return to the top of the wave with zero speed, instead of the dramatic

pitches and turns some surfers accomplish well above the wave height. A basketball rolled into a half pipe, for example, cannot rise to the opposite side of the pipe with more speed than it had at the drop-in.
Surfers apparently experience an extra burst of speed at the bottom of the wave. Somehow the surfer generates some form of thrust in the interaction of the surfboard and the wave. The surfer is extracting energy from the wave. But what is the source of this energy in the frame of the wave? We propose it is hydrodynamic lift, as experienced by sail boats and airplanes.

## IV. Hydrodynamic lift in the frame of the wave

Hydrodynamic "lift" occurs generally when a fluid flow is turned, whether or not work is done against gravity. An airplane gains lift by throwing air down, and a sail boat tacks to starboard by throwing wind to port.[8] It's ultimately the equal and opposite reaction of Newton's Third Law that provides the lift from the turning of a flow.

A neat thing about lift is that you can get it with any object in a flow. An airplane, for example, gets lift not only from the wings but also from the fuselage. In fact, any object thrown into a flow with a non-zero angle of attack will experience lift.[9] The lift does not depend on Bernoulli's principle or the lower pressure from a faster flow, as is often incorrectly used to explain lift.[10] A flat metal plate, a brick, or a paper airplane can all generate lift by turning a flow.

As a wave moves toward shore, water is constantly rolling up and over the wave. In the frame at rest with respect to the wave, the effect of the wave is to lift the water up as the wave approaches, and lower it back down as the wave passes. A surfer who has caught the wave and is moving in the frame of the wave will see a steady conveyor belt of water moving up the wave face and down over the back.

The edge of the surfboard is called the rail. At the bottom of the wave, the surfer digs the wave-side rail into the wave. The deflection of flow induced by the rail can be resolved into components horizontally and vertically along the wave face. The upward flow of the wave surface is deflected downward by the surfboard rail, and this creates an upward force which could lift the surfer up and over the wave.

There may also arise a smaller horizontal force if the surfboard is pointing slightly downward, and this force will propel the surfer forward along the wave face. To accelerate the board along its direction of motion requires that it throw extra water backwards. So how much lift can a surfer get by throwing water backward or down the wave face?

As with conventional lift, surfer lift is easily parameterized in terms of the density of water, $\rho_{water}$ , the speed of the flow past the board, $V_{board}$ , and the area of the board which is turning the flow, $A_{board}$ . The hydrodynamic pressure of any flow scales as $\rho V^2$ , and the hydrodynamic force is the product of the hydrodynamic pressure and $A_{board}$ . In aerodynamics, different objects will have a different lift coefficient multiplying these parameters. Neglecting the coefficient for now, we can write down the basic proportionality which will have units of force:

$$surfer\ lift \sim \rho_{water} V_{board}^2 A_{board} \quad . \tag{7}$$

As the wave rolls forward, water rolls up its face. If the board is parallel to the wave surface it will just slide down the wave under gravity. As the the surfer digs the rail into the wave, this downhill slide will slow and a corresponding amount of water will bounce off the board. This is like a skier carving an edge to slow the descent. If enough of the board bites into the wave, and the surfer lift exceeds the weight of the board and surfer, then the surfer and board are lifted as by a conveyor belt up and over the wave.

In deep water, the speed of the water moving up and over the wave is $V_{water} \sim \pi H / \tau$ ,[11] typically much less than the deep water wave speed. As the wave move onshore, $V_{water}$ increases while the shallow wave speed decreases. The break occurs when the speed of water moving over top of the wave approaches the wave speed.[11] Therefore we approximate $V_{board} \sim V_{wave}$ as the speed of the flow past the board for a surfer moving with the wave.

We then ask how much area of a surfboard needs to be dug into the wave to neutralize 100 kg of weight of surfer plus board. The density of sea water $\rho_{water}$ is about 1 gram per cubic centimeter. Use a modest wave speed of 10 km/hr for $V_{board}$ . Using the surfer lift equation (7) set equal to 100 kg of mass multiplied by the acceleration of gravity, the required wetted surfboard area to balance gravity is about 0.4 square meters, or about 25% of the area of a board 3 meters long and half a meter wide. Wetting more than this critical area would begin to lift the surfer up the wave. So it is entirely feasible to get a conveyor belt effect for typical surfers on typical boards on typical slow waves.

If the surfer points the board slightly down the wave but basically across the wave face, digging in just enough rail to neutralize gravity but not so much as to ride the conveyor belt up the wave, then the surfer lift is just the mass of the board plus rider multiplied by gravity,

$$surfer\ lift \sim mg \quad . \tag{8}$$

The weight of the board and surfer sets the scale for the dynamics of the deflected flow. Above this value of force accruing from the hydrodynamic pressure of the deflected flow, the surfer is taken up and over the wave face.

The momentum change of the upward flow caused by the deflection off the board is shown in Figure 1. Due to the angle $\theta$ of the board with respect to the upward flow of water over the wave, some water is deflected behind the board, and the back reaction of this horizontally deflected flow will propel the surfer forward. This turning of the flow by throwing water back propels the surfer forward just as a sail boat can be propelled by throwing air from a cross wind to stern.

Since most of the momentum transfer from the deflection is to counter gravity, then the weight of the board and surfer will set the scale of the propulsive force. When $\theta$ is zero, there is no propulsion and the deflection of flow is downward only. But with non-zero $\theta$, there is a net deflection backwards and a resultant propulsion.

It is to this forward propulsion that we look to explain the high surfer speeds. The magnitude of this propulsive force is shown in Figure 1 as $2mg\sin\theta$, and this quantifies the hydrodynamic acceleration in the frame of the wave. Although much less than the weight of the surfer because surfing angles need not be steep, this forward propulsion is persistent and would allow the surfer to accelerate to speeds much faster than available from the wave speed plus the fall of gravity. Let's see if this is big enough to account for the large surfer speeds measured with GPS.

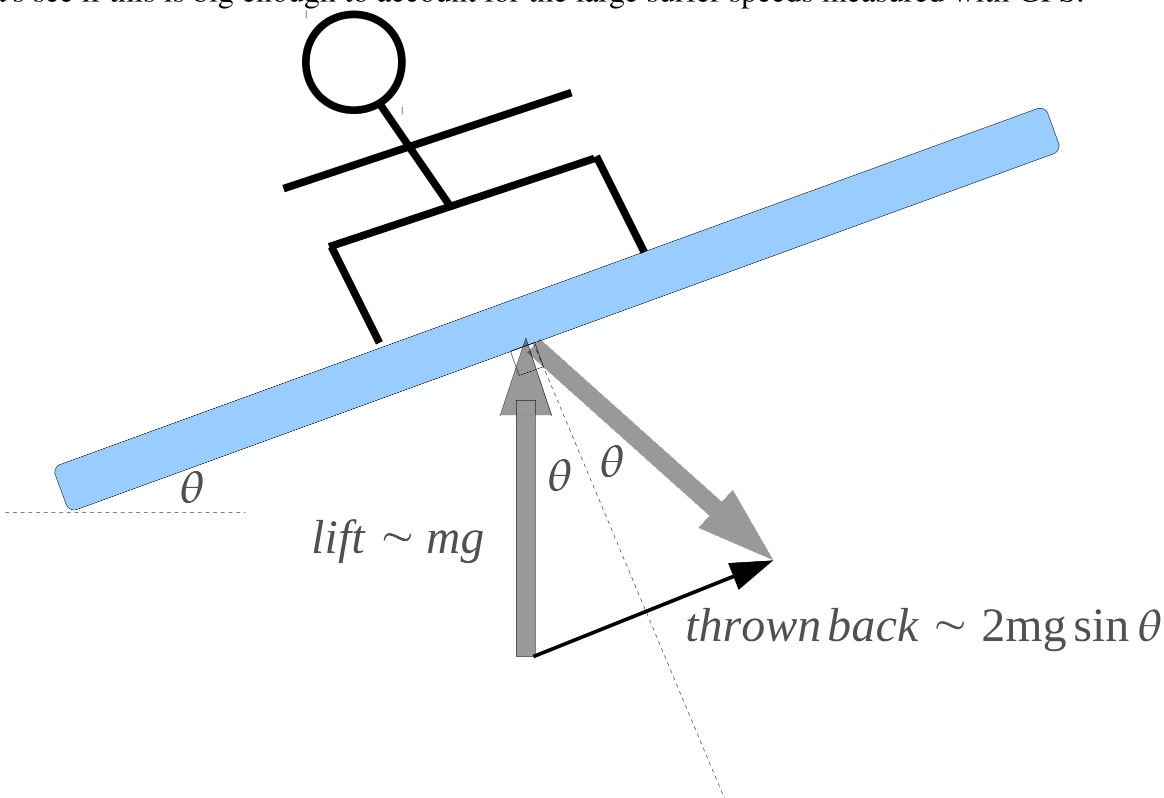


Figure 1. Flow deflection below a surfer in the frame of the wave

Consider a surfer who digs enough rail into the wave to just neutralize his weight under gravity, and also throw some fraction $\epsilon$ of his weight in water backward, and thereby generate forward propulsion. Let's give the surfer $\Delta t$ seconds of riding the wave at this acceleration. Then the surfer speed $V_s$ in the frame of the wave is simply:

$$V_s \sim \epsilon g \Delta t \quad .$$

$\epsilon$ corresponds to $2\sin\theta$ in the figure. For example, a surfboard pointed just 3 degrees down the wave as it skims across the face would produce an $\epsilon$ of 10%. A 10 second ride at 10% of $g$ would yield $V_{ss}$ = 36 km/hr. Already this value is rivaling the speed available from gravity plus wave motion of a 3 meter wave estimated in Table IV. Take a little steeper angle of attack, 6 degrees, and the speed doubles to 72 km/hr. Either dig the rail in a bit more deeply or move to a shallower angle of attack, and get an extra burst from riding the conveyor belt up the wave.

## V. Concluding Remarks

Explaining the high wave-frame surfer speeds in terms of simple hydrodynamic lift from the conveyor-belt flow of water up the wave makes the general conclusions independent of board shape. It relies only on the observation that the water rolling up the wave can easily impart a hydrodynamic pressure sufficient to support the surfer under gravity, using only a modest portion of the rail dug into the wave. Then assuming a shallow angle of attack down the wave, a significant forward propulsion is imparted to the surfer.

So hydrodynamic lift is a promising explanation for thrust in the frame of the wave. It would allow us to explain the thrust without relying on particular surfboard design parameters such as rocker, concavity, or number of fins. It would invite the general surmise that a skilled surfer can achieve hydrodynamic lift even surfing on a plank, such as an alaia.[12]

The art of surfing is the interplay between hydrodynamic lift and gravitational energy. The surfer mines energy from the wave. Ultimately, however, ocean waves are gravitational in origin and the magnitude of the effects depend on gravity. The wave speeds depend on gravity, and the hydrodynamic acceleration is a fraction of gravity.

We have ignored dissipative processes such as viscosity and friction. These idealized considerations strengthen the conclusion that wave speed and gravitational fall alone do not explain large surfer speeds, because some of the energy available from those sources will be lost in dissipation. But the idealized hydrodynamic forces calculated above will also be reduced by friction of the board with the water. Nevertheless, it appears a large and tunable amount of energy is available to the surfer to overcome these losses merely by adjusting board angle of attack or amount of rail in the wave. We should expect the surf speed record to push beyond 100 km/hr. A calculation of the top speed is the topic of another article.

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