Electromagnetic Control of Spacetime and Gravity: The Hard Problem of Interstellar Travel

L. L. Williams
Konfluence Research,
Manitou Springs, Colorado, USA

published in Astronomical Review, 6 April 2012

This review considers the hard problem of interstellar travel: overcoming fundamental limits set by the nature of space and time. The prospects for a solution to this problem are discussed in terms of the mathematical form of extensions to the classical equations of electrodynamics and general relativity, extensions which offer some prospect of faster-than-light interstellar travel with terrestrial engineering. This is tantamount to the electromagnetic control of gravity. Extensions are considered which preserve invariance under general coordinate transformations, but which relax Lorentz invariance in the limit of flat spacetimes. Such extensions describe undiscovered couplings between gravity and electromagnetism and can be understood to unify them at the classical level. Of course, only extensions consistent with past tests of Lorentz invariance are contemplated.

Among the suite of effects which arises from coupling between gravity and electromagnetism, at least two are of interest for faster-than-light travel. One is a non-Lorentzian invariant interval with its prospect of spacelike geodesics and a corresponding relaxation of the limiting speed of light. The second effect is control of the coupling constant for mass-energy to warp spacetime, which would seem to be necessary to allow terrestrial engineering of interesting space warps such as wormholes or Alcubierre warps. Both effects are mediated by as-yet-undiscovered force fields, perhaps just a single scalar field. New forces in the equations of motion and new sources of stress-energy in the Einstein equations are auxiliary effects which may be of interest for falsifying such extensions to general relativity and electrodynamics.

An example theory is presented which exhibits such extensions to the laws of gravity and electromagnetism: five-dimensional general relativity, developed between 1920 and 1960. In this theory, the faster-than-light limiting speed and the control of the coupling constant, as well as the extra forces in the equations of motion and the extra stress-energy source, all originate from a single scalar field. There is a particularly alluring identification of electric charge as a sort of momentum in the fifth dimension.

It can be perilous to speculate about undiscovered physics, but the discoveries contemplated here will apparently be necessary if our civilization, or any civilization, is to reach the stars and explore the galaxy.

CONTENTS

SCOPE OF THE REVIEW

This review discusses extensions to the equations of general relativity and electrodynamics which may be necessary if our civilization is ever to explore the galaxy. This must essentially be a problem of the electromagnetic control of spacetime and gravity.

The electromagnetic control of spacetime and gravity is a topic of general interest in physics, but particularly so for the application to interstellar travel. This is because the problem of interstellar travel is a problem of spacetime. And according to general relativity, gravity is the manifestation of the structure of spacetime. As will be made clear below, the theory of relativity places profound constraints on the practicability of interstellar travel and galactic exploration.

The interest in electromagnetic control arises because we are an electromagnetic species: our chemistry, our metallurgy, our power generation, our communications are all ultimately electromagnetic in nature. Thus, if we wish to surmount the limitations of relativity and reach the stars with any machine we construct, we may expect the machine to be ultimately electromagnetic in nature and its motion determined by its own inertia subject to electromagnetic, inertial, and gravitational forces.

Taking interstellar travel to be essentially a classical problem, the scope of this review is bounded to the classical equations of general relativity and electromagnetism. There are other approaches to overcoming the impediments of relativity and there are many reviews, e.g., [11]. Often in such approaches, however, the engineering that would achieve them is unclear, and may be unlike any conception of engineering we have today. Here, engineering feasibility is built in at the outset by considering explicitly the equations which underpin current terrestrial engineering technologies – those of classical electrodynamics and relativistic motion.

The laws of physics fall into two classes. Field equations describe how the force field behaves in the presence of material sources, and equations of motion describe how material bodies are influenced by the force fields. This review will investigate extensions to both. We start by reviewing the nature of space and time and the implied fundamental limit to interstellar travel. We then move on to considering electromagnetism in flat spacetime. Then we consider curved spacetime and the coupled equations of electromagnetism and general relativity. Then we consider the general form of as-yet-undiscovered extensions to the equations of motion and to the field equations, focusing on those aspects which offer some prospect for practical interstellar travel. A particular framework of extensions is examined, the five-dimensional theory of general relativity introduced by Kaluza [1] and subsequently developed by Jordan and colleagues [2] and independently by Thiry [3].

This review adopts the standard tensor index notation, as in, e.g., Jackson [4] or Weinberg [5]. A reader unfamiliar with the mathematical formalism used to express the equations in this review may still follow along the general points of this article.

I. FLAT SPACETIME

I.1. Motion in Spacetime

In the 20th century, we discovered that space and time are joined in a specific mathematical way – they are different aspects of *spacetime*. How you measure time, and how you describe motion in space, depends on your state of motion. The times and distances measured between events have a precise mathematical dependence on the state of motion. This mathematical dependence is called the *Lorentz transformation*. It was implicit in the laws of electromagnetism synthesized by Maxwell in the 19th century, but it was only made explicit by Einstein in 1905.

Consider any two events; say, two lightning strikes. Choose to measure the time between them, and the distance between them, with any clocks and yardsticks you please, and from any state of motion. The Lorentz transformation of the space and time intervals between events has the curious property of preserving the difference of the time and space intervals. The Lorentz-invariant *spacetime interval* is

$$c^2 d\tau^2 \equiv c^2 dt^2 - (dx^2 + dy^2 + dz^2) \qquad (1)$$

where t is the time coordinate, x, y, z are the spatial coordinates, and c is the speed of light. The Lorentz-invariant quantity τ is known as the *proper time* connecting the events. The choice of relative sign between the space and time components in (1) is immaterial, but leads to differences in sign conventions in equations such as (18). This choice is based on mathematical convenience; it is the sign difference that is crucial. Sometimes this feature is referred to as the *hyperbolic nature of spacetime*.

We can choose to formulate physical law in terms of the individual space and time coordinates, even though they are altered by the Lorentz transformation. Indeed, that's how the Lorentz-invariant Maxwell equations were originally written before Einstein. A more elegant formulation of physical law is found when written in terms of the proper time and other Lorentz-invariant quantities.

When the spacetime interval (1) is determined from the trajectory of a particle in motion, the proper time τ defined in (1), not the coordinate time t, turns out to be the fundamental parameter for the Lorentz-invariant characterization of motion. The proper time corresponds to the time coordinate of a comoving observer who naturally chooses to be at rest in his own coordinate system. Our space and time coordinates individually do not capture the essence of phenomena; we must consider the *4-velocity*

$$U^\mu \equiv \frac{dx^\mu}{d\tau} = \frac{dt}{d\tau}\frac{dx^\mu}{dt} \equiv \gamma \frac{dx^\mu}{dt} = \gamma(c, \mathbf{v}) \equiv (U^0, \mathbf{U}) \qquad (2)$$

where greek indices take on 4 values for each of the spacetime coordinates x^μ. This 4-component vector, not the usual 3-component velocity vector $\mathbf{v}$, is used in a Lorentz-invariant expression for the equations of motion. The quantity γ depends on $\mathbf{v}$ in a way which can be deduced from (1).

Because of the mathematical dependence of γ, the time component of (2) is proportional to the energy E and the spatial component is proportional to the momentum $\mathbf{p}$. Then (1) and (2) constrain the spacetime magnitude of the 4-velocity, and the associated energy and momentum:

$$c^2 = (U^0)^2 - (\mathbf{U})^2 \equiv U^\mu U_\mu = E^2/(mc)^2 - p^2/m^2 \qquad (3)$$

where the last terms are written to explicitly reflect the energy and momentum for a body of mass m. Equation (3) is the famous relativistic energy equation for particles, showing both a kinetic energy and a rest energy.

Lorentz invariance is an important concept when discussing physical law and extensions to it because this invariance constrains the mathematical form of *any* laws of physics, known or unknown, operating in the arena of spacetime. Lorentz invariance means that the form of the equations remains the same under a Lorentz transformation – the domain of special relativity. In general relativity, invariance under general coordinate transformations is considered.

I.2. Fundamental Limit to Practical Interstellar Travel

We haven't yet discussed electromagnetism or gravity but already we are prepared to discuss the fundamental limit to practical interstellar travel faced by our civilization – the hard problem. Let us emphasize that the fundamental limit is not a fuel problem; the fuel problem is the easy problem of interstellar travel. Let's assume that Project Icarus is realized, or that we have a tank of rocket fuel the size of the moon, or that we have any other unlimited amount of conventional propulsion whose ultimate purpose is to push a space ship through space. The occupants of the ship will feel the acceleration of the ship as a force throwing them to one side of the ship. Indeed, according to the theory of relativity, this acceleration will be equivalent to gravity for the ship's crew.

We want to consider the simplest possible mathematics which still captures the essence of the hard problem. Let us consider, then, the case of constant acceleration, a, irrespective of how this acceleration is engineered: chemical rocket, nuclear explosions, anti-matter, lasers, etc. This simple model is appropriate for interstellar travel because one approach is to accelerate at a constant rate for the first half of the trip, and decelerate the second half.

For acceleration and motion in the x direction, (3) implies

$$c^2 = c^2\left(\frac{dt}{d\tau}\right)^2 - \left(\frac{dx}{d\tau}\right)^2 \tag{4}$$

This provides one equation for $t(\tau)$ and $x(\tau)$, but a second is needed to determine the system. Therefore consider $dU^\mu/d\tau$.

In the frame of the space ship, $\mathbf{v} = 0$ and $\gamma = 1$. Yet they will still feel the acceleration a. So (2) implies $(dU^\mu/d\tau)_{ship} = (0, a, 0, 0)$. From this we can calculate the spacetime length of the acceleration 4-vector in the space ship frame. Since the quantity is Lorentz-invariant, it has this same value in any coordinate frame:

$$\frac{dU^\mu}{d\tau}\frac{dU_\mu}{d\tau} = -a^2 = c^2\left(\frac{d^2t}{d\tau^2}\right)^2 - \left(\frac{d^2x}{d\tau^2}\right)^2 \tag{5}$$

The simple equations (4) and (5) capture the essence of the hard problem. They have the solutions:

$$t(\tau) = \frac{c}{a}\sinh\left(\frac{a\tau}{c}\right) \tag{6}$$

$$x(\tau) = \frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right) \tag{7}$$

Equations (6) and (7) imply the well-known limiting speed of light for an object subject to unlimited acceleration:

$$\frac{dx}{dt} = \frac{at}{\sqrt{1 + a^2t^2/c^2}} \leq c \tag{8}$$

This limit would preclude practical insterstellar travel for our civilization because the distances between the stars are so vast. Our own galaxy is 100,000 light years in diameter, and the mean distance between stars is about 3 light years. Thus, to cross our galaxy would require 100,000 years on the home planet. The galaxy could not be explored by any civilization which persists for a time less than this.

The equations (4) and (5) also imply the well-known effect of time dilation as expressed in (6). A clock on board the space ship measuring an interval of proper time τ would be extremely time-dilated relative to a clock at rest in the home civilization. During the time a spacecraft accelerated at 1 g for 5 years as measured on board, 74 years would pass on the home planet. Yet in this time the travelers could scarcely reach the nearest stars.

Under constant acceleration, the spaceship speed quickly approaches the speed of light. Equation (8) shows that the spaceship can reach half the speed of light by holding 1 g of acceleration for half a year. Such relativistic speeds would dilate the clocks of space travelers enough for a spaceship to travel an arbitrary distance during the lifetime of its crew, but the civilization which sent them can never be informed of their discoveries because it would evolve out of existence in that time. Travelers moving fast enough to reach the stars become temporally detached from their home civilizations and can never return to them.

So the hyperbolic nature of spacetime (1) as expressed in our current understanding of the principles of special relativity sets two basic limits on practical interstellar travel. One is the limiting speed of light, combined with the vast distances to the stars, will prevent a civilization from exploring any significant fraction of the galaxy before the civilization evolves out of existence. The second basic limit is the effect of time dilation, which will temporally disconnect any astronaut from her civilization even though the same time dilation effect would allow the astronaut to go an arbitrary distance in her lifetime.

If our civilization is to colonize any part of the galaxy, it would require some way to break the light barrier and accomplish faster-than-light travel. This is a profound dream because (1) also implies that faster-than-light travel is tantamount to time travel. If faster-than-light travel is possible, we may expect any machine which could achieve this would be electromagnetic in nature for the reasons given in the introduction. Certainly there are other approaches to investigating departures from the known laws of physics; here we wish to explore linkages between gravity and electromagnetism. Let us turn, then, to consider extensions to the laws of spacetime and electromagnetism which may allow us to surmount the light barrier through the electromagnetic control of gravity and spacetime. We start by considering the known laws of gravity and electromagnetism.

I.3 Motion in Electromagnetic Fields

The momentum equation for a body of electric charge q in a combined electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ is given by the Lorentz force law (here in cgs units):

$$m\frac{d\mathbf{U}}{dt} = q\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right) \tag{9}$$

The energy equation for such a body is

$$mc\frac{dU^0}{dt} = q\mathbf{v} \cdot \mathbf{E} \tag{10}$$

Equations (9) and (10) are not separately Lorentz invariant but they can be combined using (2) in the single invariant equation of motion for a charged body in an electromagnetic field:

$$m\frac{dU^\mu}{d\tau} = \frac{q}{c}F^{\mu\nu}U_\nu \tag{11}$$

where $F^{\mu\nu}$ is the electromagnetic field strength tensor. It has 6 independent components – the vector components of $\mathbf{E}$ and $\mathbf{B}$. For a full development of (11), see, e.g., [4].

I.4. Field Equations of Electromagnetism

The field equations of electromagnetism are the Maxwell equations. They are commonly written in terms of the electric and magnetic field vectors (here in cgs units):

$$\nabla \cdot \mathbf{E} = 4\pi\rho \quad , \quad \nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} \tag{12}$$

$$\nabla \cdot \mathbf{B} = 0 \quad , \quad \nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \tag{13}$$

where ρ is the electric charge density and $\mathbf{J}$ is the electric current vector. These equations provide a nice example of *coupled* field equations: each field can act as a source for the other. Equation of (12) obviously involves both the magnetic field and electric field. In the subsequent development we will look for analogous couplings expressed in the equations for the electromagnetic and gravitational fields.

The classic form (12) and (13) is useful for engineering purposes but it is not Lorentz invariant. The magnetic and electric field vectors do not themselves have the proper transformation properties, but they do when considered as the components of the tensor $F^{\mu\nu}$. This connection is made in terms of the standard electric and magnetic potentials:

$$\mathbf{E} = -\nabla\Phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} \quad , \quad \mathbf{B} = \nabla \times \mathbf{A} \tag{14}$$

Equations (14) satisfy (13) identically. Equations (12) then follow from:

$$\partial_\nu F^{\nu\mu} = \frac{4\pi}{c}J^\mu \tag{15}$$

where $F^{\nu\mu} \equiv \partial^\nu A^\mu - \partial^\mu A^\nu$ and where coordinate partial derivatives are abbreviated $\partial/\partial x^\nu \equiv \partial_\nu$.

The form of (15) manifests the Lorentz invariance of the Maxwell equations. The electromagnetic current 4-vector $J^\mu \equiv (\rho c, \mathbf{J})$ comprises the 3 spatial components of electric current, and the one temporal component of electric charge density. The electromagnetic potential 4-vector $A^\mu \equiv (\Phi, \mathbf{A})$ comprises the electric potential and the magnetic vector potential.

At first glance, (15) would appear to be 4 equations in the 4 unknowns A^μ. However, conservation of charge requires that $\partial_\mu J^\mu = 0$, which constrains the 4 equations. This constraint reduces the number of independent equations to 3. The extra degree of freedom is fixed by the choice of a *gauge* – this is the famous *gauge invariance* of electromagnetism.

For a full development of (15) and discussion of its Lorentz invariance and gauge invariance, see, e.g., [4].

I.5. The Prospects of Electromagnetic Forces in Flat Spacetime

The Lorentz force law (11) does not offer any prospect for overcoming the limits to interstellar travel described in (I.2). An arbitrarily large electric field can be imagined in the momentum equation (9) but it will not change the basic nature of the solution described in (8) and (6). The magnetic field offers no prospect either because it will act only to deflect motion of a body without changing its energy. The Maxwell equations (15) provide no prospect because any electric or magnetic fields we may generate in accordance with them will still be limited in their effects on the motion of material bodies as just described. We can generate arbitrarily large and complex electromagnetic fields, but those fields cannot be used to overcome the limits on interstellar travel set by the nature of spacetime.

So any machine we may build based on electromagnetic forces acting in flat spacetime described by (1) could not overcome the fundamental limits to interstellar travel. Let us turn to consider physics in curved spacetime.

II. CURVED SPACETIME

Consider now the properties of spacetime described in Einstein's theory of general relativity. General relativity extends the principle of Lorentz invariance, observed to hold in the flat spacetime of special relativity, to the principle of *general covariance*: the equations of general relativity are required to preserve their form under an arbitrary coordinate transformation. We use the term "Lorentz invariant" to refer to equations which are invariant under Lorentz transformations, and "covariant" to refer to equations which are invariant under arbitrary coordinate transformations. The equations of general relativity are expressed in terms of scalars, vectors, and tensors which have well-defined coordinate transformation properties.

II.1. Motion in Gravitational Fields

To treat motion in gravitational fields requires a generalization of (1):

$$c^2 d\tau^2 \equiv g_{\mu\nu} dx^\mu dx^\nu \tag{16}$$

where $g_{\mu\nu}$ is identified with the *metric* describing the covariant interval between spacetime events. It is clear from the commutativity of the coordinate length elements in (16) that $g_{\mu\nu}$ must be symmetric in μ and ν. For the 4 dimensions of spacetime, this implies $g_{\mu\nu}$ has 10 components. Equation (3) still holds in that $U^\mu U_\mu = c^2$.

In the absence of gravitational fields, $g_{\mu\nu}$ is diagonal and constant, and (16) reduces to (1). This is the basic distinction between special relativity and general relativity, and is also the distinction between flat spacetime and curved spacetime. Furthermore, it is an axiom of general relativity that coordinates can always be chosen locally so that (1) holds.

The equation of motion for any particle (even a massless one) in a gravitational field is given by *the geodesic equation*, the relativistic generalization of Newton's law of motion in a gravitational field:

$$U^\alpha \nabla_\alpha U^\mu \equiv U^\alpha(\partial_\alpha U^\mu + \Gamma^\mu_{\alpha\beta} U^\beta) = \frac{dU^\mu}{d\tau} + \Gamma^\mu_{\alpha\beta} U^\alpha U^\beta = 0 \tag{17}$$

$\Gamma^\mu_{\alpha\beta}$ is *the affine connection*, a quantity which is formed from derivatives of the metric tensor $g_{\mu\nu}$. ∇_α is *the covariant derivative*. For a full development of (17), see, e.g., [5].

The effects of gravity are built into the covariant derivative because $\Gamma^\mu_{\alpha\beta}$ is effectively the gravitational field and $g_{\mu\nu}$ is the gravitational potential. Because $g_{\mu\nu}$ has this dual identity as the gravitational potential (17) and the spacetime metric (16), gravity is identified with the curvature of spacetime. We use the terms spacetime and gravity interchangeably in this review.

II.2. Field Equations of Gravity

The field equations for $g_{\mu\nu}$ are called the Einstein equations, and are always written in a manifestly covariant tensor formulation,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu} \tag{18}$$

where $R_{\mu\nu}$ and R are rather complicated functions of the metric tensor, $g_{\mu\nu}$. The *stress-energy tensor* is $T_{\mu\nu}$, which constitutes a source term to the field equations similar to the source J^μ in (15). The Newtonian gravitational constant is G. The constant multiplying the stress-energy tensor is very small, which expresses the relative weakness of the gravitational force. It's why a planet-sized amount of matter is necessary to get significant gravity. The term in Λ is the cosmological constant, and corresponds to energy in the vacuum. For a full discussion and development of (18), see, e.g., [5].

Unlike the Maxwell equations (12) and (13), which were synthesized from centuries of observations of electric and magnetic effects, the Einstein equations (18) were obtained purely theoretically from considerations of general covariance and matching Newton's law of gravity in an appropriate limit. While the predictions of (18) are confirmed many times over, its construction retains an element of artistry. The LHS of (18) has a clear, turn-the-crank prescription in terms of the metric tensor, but the RHS is ad hoc. Einstein himself called this distinction between the two sides of (18) "marble" and "wood".

A suitable stress-energy tensor can be constructed for any source, e.g., massive particles, fluids, or pure radiation. The stress-energy tensor is always constructed so that its covariant divergence vanishes, $\nabla_\mu T^{\mu\nu} = 0$, and this vanishing divergence represents energy and momentum conservation. This conservation property used in construction of the stress-energy tensor is mirrored by the vanishing of the covariant divergence of the "marble" side: $\nabla_\mu(R^{\mu\nu} - Rg^{\mu\nu}/2) = 0$. These 4 equations are called the *Bianchi identities* and are satisfied identically for any $g_{\mu\nu}$. Naively one may expect that (18) are 10 independent equations in the 10 unknowns $g_{\mu\nu}$. But the 4 Bianchi identities reduce this number to 6 independent equations. The other 4 equations necessary to specify $g_{\mu\nu}$ come from the choice of spacetime coordinates. This coordinate invariance is the analog of gauge invariance for (15).

While the Maxwell equations (15) are linear in the fields, the Einstein equations (18) are not. This means the Einstein equations do not allow any analytic prescriptions for calculating general solutions. However, many exact solutions to the Einstein equations have been discovered, such as the Schwarzschild solution, the relativistic generalization of Newton's law of gravity that describes black holes.

II.3. Coupling between Gravity and Electromagnetism

Before we move to consider couplings yet undiscovered, let us consider first the known coupling between electromagnetism and gravity. The Einstein equations (18) can be solved under conditions of an electromagnetic stress-energy tensor. Electromagnetic energy can be a source of spacetime curvature just as mass can:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T^{EM}_{\mu\nu} \tag{19}$$

where the electromagnetic stress-energy tensor is

$$T^{\alpha\beta}_{EM} = \frac{1}{4\pi} \left(g^{\alpha\mu} F_{\mu\lambda} F^{\lambda\beta} + \frac{1}{4} g^{\alpha\beta} F_{\mu\lambda} F^{\mu\lambda} \right) \tag{20}$$

Because the gravitational potential $g_{\mu\nu}$ is so intimately tied to the properties of spacetime, the *Principle of General Covariance* which underpins general relativity provides a simple prescription for writing the field equations of electromagnetism in gravitational fields. They are obtained from the flat-spacetime Maxwell equations (15) by replacing the partial derivative ∂_μ with a covariant derivative ∇_μ:

$$\nabla_\nu F^{\nu\mu} \equiv \partial_\nu F^{\nu\mu} + \Gamma^\nu_{\nu\alpha} F^{\alpha\mu} + \Gamma^\mu_{\nu\alpha} F^{\nu\alpha} = \frac{4\pi}{c} J^\mu \tag{21}$$

The terms multiplying $\Gamma^\nu_{\alpha\beta}$ and $F_{\alpha\beta}$ represent the effects of gravity on electromagnetic fields.

Likewise, a gravitational field can deflect electromagnetic radiation, as in the classic test of the bending of starlight by the sun. The full equations of motion in combined electromagnetic and gravitational fields is obtained from (11) and (17)

$$\frac{dU^\mu}{d\tau} + \Gamma^\mu_{\alpha\beta} U^\alpha U^\beta = \frac{q}{mc} F^{\mu\nu} U_\nu \tag{22}$$

where the electromagnetically-induced gravitational force $\Gamma^\mu_{\alpha\beta}$ is obtained from (19).

Equation (21) is an example of the modification of an equation discovered earlier, (15), with terms whose effects were negligible when (15) was first discovered. We now understand (15) as the limiting case of (21). We will consider additional extensions in our later development.

II.4 The prospects of electromagnetic forces in curved spacetime

Some solutions of the Einstein equations which have implications for interstellar travel have been found, solutions which offer the promise of surmounting the light barrier. The Einstein equations (18) or (19) allow for distortions in spacetime, *wormholes*, which can connect spatially disparate parts of the galaxy and which therefore could in principle be traversed in arbitrarily short times [6]. (Such wormholes need not be generated electromagnetically). Wormholes are extreme deformations of spacetime, similar in some ways to black holes, at different locations in the galaxy. They could feasibly be "connected" to each other. A traveler would descend into one wormhole and pop out in the other a short time later but very far away. So the discovery of curved spacetime offers a way to accomplish faster-than-light travel in principle. Such superluminal travel is still prohibited locally, but spacetime itself can be curved to connect different points in space.

We already have an engineering problem because even if a civilization had the wherewithal to engineer such structures, the civilization would presumably still have to travel sub-luminally across the galaxy to the "exit" point and build it. It is sort of like the pioneers hoping to take trains or airplanes across North America; they would still have to travel by covered wagon to the destination and build the airport or the subway exit.

There is another approach to interstellar travel which leverages curved spacetime. Alcubierre [7] discovered a "warp drive" solution to the Einstein equations in which a specially engineered spacetime bubble can transport its occupants superluminally across the galaxy. Like the wormhole solutions, spacetime only allows locally subluminal travel, but in this case general relativity allows a bubble of flat space to move superluminally. This at least is better than the wormhole approach because one need not cross the galaxy first sub-luminally to build the transport system.

Unfortunately, both the wormhole and the Alcubierre solutions would require the engineering of astronomical amounts of mass-energy, and even exotic *negative energy*, to warp spacetime into the desired configurations of 100-meter size wormhole throats or warp bubbles [8]. The root of this impractibility is the very small number G/c^4 which provides the coupling between energy and spacetime curvature in (18) and (19). So while feasible within the context of our understanding of physical law, wormholes and Alcubierre drives are impractical for the solar-mass or Jupiter-mass engineering and exotic energies which would be required to realize them. These solutions to the Einstein equations give us hope that faster-than-light travel is not impossible in principle, but they have no practical realization yet for terrestrial engineering. While there is some intriguing work being done on bringing wormholes or Alcubierre warps down to terrestrial scale, so far we have no way to achieve it through terrestrial engineering. In order to make interstellar travel feasible for a civilization existing for a limited time around some star, qualitatively new extensions to the known laws of physics will be required.

If our civilization is to advance to the stars in any machine we may build with our current electromagnetically-based technologies, we must discover some additional coupling between spacetime and electromagnetism which could allow us to overcome the light barrier with terrestrial-scale engineering. We expect it to ultimately be in terms of some extension to special relativity and our current understanding of the nature of spacetime.

III. BEYOND SPACETIME

III.1. Covariant Extensions of Physical Law

In our search for possible new couplings between spacetime and electromagnetism, we are guided by the principle of general covariance: that any equation of physical law must preserve its form under a general coordinate transformation. An expression of the fundamental nature of spacetime, irrespective of the forces and fields, this principle has been applied with great success across all fields of physical law since its introduction by Einstein over a century ago. While there is no "relativity force" per se, as there is an electric force, for example, the application of the principle of general covariance has yielded our understanding of gravity as a coordinate-dependent phenomenon. We therefore continue to enforce general covariance as a discriminator for any new theory.

Yet, as described above, it is considerations of special relativity and Lorentz invariance (1) which set the limits on practical interstellar travel that we wish to surmount. We require a sophisticated extension of physical law which is informed by our understanding of Lorentz invariance, but gets around our current understanding of the limitations described in section I.2. So we allow some relaxation of the Lorentz coordinate transformation per se, but still demand that the laws of physics retain their mathematical form under the more general coordinate transformation we wish to contemplate.

As we consider these extensions to physical law, we keep in mind that we are not overthrowing the Maxwell or Einstein equations, but instead we are seeking a new regime of operation which has not been experimentally accessed before. For example, the original equations synthesized by Maxwell (15) are now understood to include extra general-relativistic effects described in (21). These effects were not seen by Maxwell because they are negligible on earth's surface where Maxwell's equations were first discovered; (15) is understood to be the limiting case of (21).

So in this spirit we contemplate a relaxation of invariance under the Lorentz transformation, and with it the limiting speed of light. We saw how the fundamental limit to interstellar travel arises from (1). To surmount the light barrier would presumably require an extension to (1) and its generalization (16):

$$g_{\mu\nu} dx^\mu dx^\nu = c^2 d\tau^2 \pm d\Omega^2 \tag{23}$$

where Ω is some yet-to-be-discovered function of the coordinates x^μ. We choose to write this function suggestively as a differential to accord with the general form of (16). We dispense with any multiplicative correction to (16) because the terms in $dx^\mu dx^\nu$ would be indistinguishable from an arbitrary metric. We therefore suppose the extension (23) to (16) must be additive.

The implications of (23) are profound. At first it may appear that (23) is mathematically equivalent to (16). The important distinction is that Ω does not vanish when $g_{\mu\nu}$ reduces to the flat spacetime metric and (16) reduces to (1). Consider the variation in (4) implied by (23):

$$c^2 \pm \left(\frac{d\Omega}{d\tau}\right)^2 = c^2 \left(\frac{dt}{d\tau}\right)^2 - \left(\frac{dx}{d\tau}\right)^2 \tag{24}$$

We have not specified the functional form of Ω, nor ruled in or out any particular behaviors. But comparison with (4) makes it clear the equations now support the necessary degrees of freedom to contemplate spacelike geodesics and faster-than-ligth travel, and dramatically alter the limit described in section I.2.

Of course, the search for Lorentz violations has been under way for a century through a variety of different experiments. No violation has been seen yet and there is no evidence for Lorentz violations in the environments tested by the various experiments. So if they exist at all, Lorentz violations must be in a test regime that so far has not been explored – perhaps a precisely engineered variation of known or unknown force fields, but a variation that does not occur under natural conditions. An example of emergent forces of this type is the vacuum fluctuation forces of the Casimir effect; they are not felt by natural macroscopic systems unless a machine is built to express them.

We also want to consider the electromagnetic control of gravity, extensions to (18) which would obviate the need for astronomical amounts of mass or energy to warp spacetime into wormholes or Alcubierre drives. Let us therefore consider some new control of the coupling constant G/c^4 multiplying the stress-energy tensor:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} \Psi T_{\mu\nu} + \Theta_{\mu\nu} \tag{25}$$

where Ψ is a function of the coordinates x^μ. The function $\Theta_{\mu\nu}$ constitutes some yet-to-be-discovered source of spacetime curvature. The cosmological constant term is dropped from (18) but it could be considered implicit in

$\Theta_{\mu\nu}$. Two new fields, Ψ and $\Theta_{\mu\nu}$, are introduced to the Einstein equations; they are jointly constrained by the Bianchi identities.

For extensions to the Maxwell equations, there is no need to contemplate adjustments to the coupling constant on the RHS of (21) as we did with Ψ in (25) – the electromagnetic forces are already quite strong and easy to generate through terrestrial engineering. But we do allow another source of electromagnetic fields Υ^μ:

$$\nabla_\nu F^{\nu\mu} = \frac{4\pi}{c} J^\mu + \Upsilon^\mu \tag{26}$$

Finally, additional forces Ξ^μ are contemplated in the equations of motion (22):

$$\frac{dU^\mu}{d\tau} + \Gamma^\mu_{\alpha\beta} U^\alpha U^\beta = \frac{q}{mc} F^{\mu\nu} U_\nu + \Xi^\mu \tag{27}$$

Although they have been introduced separately, the quantities Ω, Ψ, $\Theta_{\mu\nu}$, Υ^μ, and Ξ^μ, are expected to be related by new, invariant field equations, as well as by the constraints of charge conservation and the Bianchi identities. So the number of degrees of freedom are going to be smaller than one might naively expect from independently adding new terms to the field equations and equations of motion. Indeed, as we will see when we consider a specific theory in the next section, a single yet-to-be-discovered scalar field is sufficient to account for all these effects.

Of the generalized extensions (23), (25), (26), (27), perhaps (25) has been the most studied. The classic Brans-Dicke extensions of general relativity as described by [5] are of the form (25). Brans-Dicke theory posits a scalar field Ψ in addition to the standard metric $g_{\mu\nu}$, and the Brans-Dicke scalar field equation has matter as its source. The stress tensor $\Theta_{\mu\nu}$ is a function of Ψ, but Brans-Dicke theory assumes the scalar field does not enter the equations of motion: $\Xi^\mu = 0$.

More recent considerations of (25) stem from late-20th-century discoveries in cosmology. Specifically, the acceleration of the Hubble expansion is modeled mathematically as a cosmological constant in (18) [12]. Considerations of quantum theory suggest that there is a vacuum energy which would also manifest as a cosmological constant. And the standard model of cosmology has an early era of rapid inflation which is modeled mathematically with a scalar field that manifests in (25) as $\Theta_{\alpha\beta}$ [12].

Although a new scalar field Ψ would not be unheard of, any discovery of a new force Ξ^μ in the equations of motion or a new source of electromagnetic fields Υ^μ would be truly revolutionary. Let us now consider a specific theory which illustrates the sort of extensions considered above.

III.2. The Kaluza Unification of Gravity and Electromagnetism

Soon after Einstein's completion of general relativity, Kaluza [1] introduced a theory which unified general relativity and classical electromagnetism. This was done by writing the Einstein equations and the geodesic equation in 5 dimensions using the five-dimensional (5D) metric $\widetilde{g}_{ab}$ where:

$$\widetilde{g}_{\mu\nu} = g_{\mu\nu} - k^2\phi^2 A_\mu A_\nu \quad , \quad \widetilde{g}_{5\nu} = -k\phi^2 A_\nu \quad , \quad \widetilde{g}_{55} = -\phi^2 \quad , \quad k^2 = 16\pi G/c^4 \tag{28}$$

Here greek indices are continued to be used to span the 4 dimensions of spacetime, the index 5 signifies the fifth dimension, and roman indices span all 5 dimensions. A tilde is used to denote 5D quantities. Thus, seen as a matrix, the 5D metric $\widetilde{g}_{ab}$ consists of the 4D metric $g_{\mu\nu}$ "framed" by the electromagnetic potential A^μ and a new scalar field ϕ at the 5th diagonal.

This theory has a 5D invariant length element $ds^2 \equiv \widetilde{g}_{ab} dx^a dx^b$. From (28) one immediately obtains the extension similar to (23):

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu - \phi^2 (k A_\nu dx^\nu + dx^5)^2 \tag{29}$$

Note that comparison of (28) or (29) with (1) shows that the 5th dimension has a spacelike signature. To reproduce standard 4D physics requires the 5th dimension have the same sign in the metric as the space coordinates.

A key assumption of the Kaluza theory is *the cylinder condition*, which is that none of the fields depend on the 5th coordinate: $\partial_5 \widetilde{g}_{ab} = 0$. The cylinder condition is partially out of convenience: it enormously simplifies the theory, yet still provides some non-trivial predictions not found in 4D physics. Furthermore, the 5th dimension is mysterious and the meaning of variation in the 5th dimension is difficult to fathom. Still, the ad hoc nature of the cylinder condition is perhaps the main weakness of the theory.

The Kaluza field equations for $g_{\mu\nu}$, A^μ, and ϕ are then obtained from the 5D vacuum Einstein equations

$$\widetilde{R}_{ab} - \frac{1}{2}\widetilde{g}_{ab}\widetilde{R} = 0 \tag{30}$$

applied to (28) under the constraint of the cylinder condition, resulting in a set of extensions to physical law similar to (25) and (26):

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} \phi^2 T^{EM}_{\mu\nu} + T^\phi_{\mu\nu} \tag{31}$$

$$\nabla^\mu F_{\mu\nu} = -3F_{\mu\nu}\partial^\mu \ln\phi \tag{32}$$

$$\nabla_\alpha \nabla^\alpha \phi = \frac{4\pi G}{c^4}\phi^3 F_{\alpha\beta}F^{\alpha\beta} \tag{33}$$

$T^\phi_{\mu\nu}$ is a stress-energy tensor for the scalar field whose precise functional dependence on ϕ will not concern us here. Thus the vacuum 5D theory provides the Einstein equations for $g_{\alpha\beta}$, the Maxwell equations for A^α, and a field equation for ϕ – an elegant unification.

Kaluza originally assumed ϕ was constant. In this limit, (31) reduces to (19) and (32) reduces to (21). However, this assumption restricts the electromagnetic field through (33). Only much later did Jordan [2] and Thiry [3] independently obtain the full set of self-consistent field equations (31), (32), and (33).

If the cylinder condition is relaxed, an enormous number of degrees of freedom are made available, and it's not at all clear how to interpret the additional terms comprising $\Theta_{\mu\nu}$ in (25). Overduin & Wesson [9] suggest an interpretation of the x^5 derivatives in terms of material sources such as $T_{\mu\nu}$ and J^μ. In this way, matter and forces are both obtained purely from geometry; only marble, no wood.

The 4D equations of motion in this theory are obtained from a 5D geodesic equation:

$$\widetilde{U}^b \widetilde{\nabla}_b \widetilde{U}^\mu = \frac{d\widetilde{U}^\mu}{ds} + \widetilde{\Gamma}^\mu_{ab}\widetilde{U}^a\widetilde{U}^b = 0 \qquad \Longrightarrow$$

$$\frac{dU^\mu}{d\tau} + \widetilde{\Gamma}^\mu_{\alpha\beta}U^\alpha U^\beta + 2\widetilde{\Gamma}^\mu_{5\alpha}U^\alpha U^5 + \widetilde{\Gamma}^\mu_{55}(U^5)^2 + U^\mu \frac{d}{d\tau}\ln\left(\frac{cd\tau}{ds}\right) = 0 \tag{34}$$

where $\widetilde{U}^a \equiv dx^a/ds$ and $U^a \equiv dx^a/d\tau$.

To make contact with the standard equations of motion (22) requires the identification

$$kU^5 \equiv k\frac{dx^5}{d\tau} = \frac{q}{mc} \tag{35}$$

In other words, electric charge arises from "motion" in the 5th dimension. With this identification and (34) we can write the extensions Ξ^μ to the equations of motion introduced in (27)

$$\Xi^\mu_{\text{Kaluza}} = -\frac{q}{mc}U^\alpha A_\alpha\ \partial^\mu\phi^2 - \frac{(q/m)^2}{32\pi G}\partial^\mu\phi^2 + U^\mu\frac{d}{d\tau}\ln\left(\frac{cd\tau}{ds}\right) + O(k^2A^2) \tag{36}$$

This equation is correct to first order in the small dimensionless quantity $kA << 1$. Because k as given by (28) is so small, even an astronomical electromagnetic field such as the 10^{12} Gauss magnetic field of a neutron star would still have $kA \sim 10^{-6}$. When the scalar field is constant, (36) reduces to only the second-order terms.

Note from (35) and (28) that the gravitational constant sets a universal charge-to-mass ratio: it has units of q^2/m^2. Sub-atomic particles such as electrons and protons have large charge-to-mass ratios when expressed in units of $G^{1/2}$. For such particles, the second term on the RHS of (36) could be quite large; the manifestation of these forces is controlled by the variation of the scalar field. Furthermore, such large charge-to-mass ratios imply spacelike 5D intervals in (29).

III.3 Feasibility Discussion

Let us now summarize the general picture of electromagnetic control of gravity developed in the last 2 sections. We began with the observation that our current understanding of physical law limits the ability of our civilization to explore the galaxy. With a desire to search for new effects which could allow interstellar exploration with terrestrial engineering, extensions to the laws of general relativity and electrodynamics are considered. Such extensions are tantamount to the electromagnetic control of gravity.

As an example of a theory of the electromagnetic control of gravity, the classical, 5D Kaluza theory under the cylinder condition provides for new electromagnetic forces (36) from an as-yet-undiscovered scalar force field ϕ. The new force is electromagnetic in that it couples to electric charge; note how the first two terms in (36) multiply the electric charge. The scalar field is in turn coupled to the electromagnetic field $F_{\alpha\beta}$ through (32) and (33); each is a source in the field equation of the other. The scalar field acts to modulate the coupling of mass-energy to spacetime curvature through (31). Finally, these scalar-electromagnetic effects can modify the invariant interval through (29).

The latter two qualities are of interest for interstellar travel. A modulation of the gravitational constant controlling the amount of energy required to warp spacetime (31) would be the sort of discovery necessary to make wormholes or Alcubierre warps feasible with terrestrial engineering. And a non-Lorentzian adjustment to the invariant length element (29) could portend a change in the hyperbolic structure discussed in section I.2. The new forces in (36) may not have implications for faster-than-light travel, but they would be perhaps measurable and diagnostic of the general structure of the theory.

Perhaps the most fascinating aspect of the Kaluza theory is the intepretation of electric charge as motion in the fifth dimension. Electric charge is the fifth component of an energy-momentum-charge 5-vector [10]. This means electric charge is not a Lorentz scalar in this theory. But the cylinder condition requires it behave like a Lorentz scalar. Since the 5D metric does not depend on the fifth coordinate, there is a conserved quantity $\widetilde{g}_{5a}\widetilde{U}^a \propto (q + 16\pi G m A_\nu U^\nu / c^3)\phi^2$. In the absence of electromagnetic fields and with constant ϕ, the charge q is invariant. With electromagnetic fields, however, there can be minute variations in charge but it is probably unobservable [10]. Thus the cylinder condition accounts for why charge behaves as a scalar.

The Kaluza theory provides a particularly simple illustrative example of extended physics because all the effects and couplings are achieved with a single new, as-yet-undiscovered scalar field. One could build more-complicated expressions for Ω, Ψ, $\Theta_{\mu\nu}$, Υ^μ, and Ξ^μ from more-elaborate tensor fields, but it's useful to consider first the simplest extensions to physical law. The scalar-only theory could also be treated as a parameterization of other theories.

We may ask what the prospects are for the universe harboring a scalar field which has gone undiscovered so far. The answer is: quite good. There are unsolved problems in cosmology which could feasibly be addressed by a scalar field and associated scalar force. The era of inflation in the standard big bang model is expressed in terms of a scalar field. There is also a search underway for a quantum scalar field – but this is not currently expected to have long-range force effects. None of these motivations for a scalar field were known during the 1920s when the Kaluza theory was originally considered and discarded. There is some similarity between the Kaluza scalar field and the Brans-Dicke scalar field, in that they both modify the Einstein equations in the same way (31). But the Brans-Dicke scalar field has no coupling to charge or electromagnetic fields, and does not appear in the equations of motion.

To consider the quantitative feasibility of this particular theory, let us return to (36) to ask what sort of gradients in the scalar field are needed to get an observable effect in (27). The field equation (33) provides a lengthscale for the variation of ϕ. For a neutron star magnetic field of 10^{12} Gauss, the lengthscale is of order one astronomical unit [10]. Of course, the variation induced in ϕ from terrestrially-engineered electromagnetic fields will be negligible – the scale of variation would be the size of the universe. Just as the small coupling constant G/c^4 in (19) requires astronomical amounts of mass-energy to curve spacetime, the same small coupling constant in (33) requires astronomical amounts of electromagnetic energy to "put a dent" in the scalar field. This may be troubling for our hope to use electromagnetic means to engineer the scalar field.

In this theory, however, the gradient of the scalar field, which is small, is multiplying a term which could be quite large: the second term on the RHS of (36). For a proton, the quantity $q/kmc \sim 10^{21}c$ [10]. Although this is quite large, it is modulated by very small gradients in the scalar field which serve to keep small the modification to the equations of motion. Further investigation of (36) is needed.

CONCLUSION

The Kaluza theory may not be the correct theory to extend the laws of gravity and electromagnetism in a way that could make interstellar exploration feasible for our civilization. But it does show us the sorts of behavior we may expect from such a theory if it exists: new couplings between gravity and electromagnetism would manifest as a correction to the invariant interval of general relativity, and manifest as new forces in the equations of motion. The coupling constant for mass-energy to warp spacetime is controlled through a scalar field. We have not provided solutions here which unequivocally prove that such effects are possible, but only that the theory has the mathematical degrees of freedom to accommodate such effects. In any correct theory, the new forces must be small or otherwise operative in a regime that has so far not been tested experimentally – otherwise we would have discovered them already. Thus, the sort of extensions to physical law hypothesized here will likely be falsifiable, which is desirable in any such theory. Indeed, the 5D theory continues to attract the attention of researchers around the world. As we interpret such experiments we must keep in mind that just a single scalar field added to the existing known fields of gravity and electromagnetism could account for all these effects; more complicated tensor fields are not mathematically necessary. With discoveries in cosmology indicating a new scalar field, discovery of electromagnetic control of gravity remains a possibility.

GLOSSARY OF SYMBOLS

A^μ is the electromagnetic potential (4 numbers)
c is the speed of light
∇_α is the covariant derivative
$F_{\mu\nu}$ is the electromagnetic force field (6 numbers)
$g_{\mu\nu}$ is the gravitational potential and the spacetime metric (10 numbers)
$\Gamma^\mu_{\alpha\beta}$ is the gravitational force field (40 numbers); affine connection; Christoffel symbol
G is the gravitational constant
J^μ is the electric charge and electric current (4 numbers)
Λ is the cosmological constant
m is the mass of a material object
∂_α is the partial derivative
q is the charge of a material object
$R_{\mu\nu}$ is a complicated set of derivatives of $g_{\mu\nu}$ (10 numbers); the curvature of spacetime
$R \equiv R_{\mu\nu} g^{\mu\nu}$
U^μ is the 4-velocity of an object moving in spacetime (4 numbers)
s is the 5D invariant interval
τ is the 4D invariant interval; proper time
$T_{\mu\nu}$ is the energy and momentum fluxes of matter and/or radiation (10 numbers)
x^μ is the spacetime coordinate (4 numbers)

Ω is a hypothetical extension of the spacetime interval
Ψ is a hypothetical field which controls the coupling of mass-energy to gravity
$\Theta_{\mu\nu}$ is a hypothetical new source of spacetime curvature (10 numbers)
Υ^μ is a hypothetical new source of electromagnetic fields (4 numbers)
Ξ^μ is a hypothetical new force field (4 numbers)

[1] T. Kaluza, *Sitzungsberichte der K. Preussischen Akademie der Wissenschaften zu Berlin*, 966 (1921).
[2] P. Jordan and C Müller, *Zeitschrift Fur Naturforschung*, **2a**, 1-2 (1947); P. Jordan, *Schwerkraft und Weltall*, (Vieweg, Braunschweig),(1955); P. Jordan, *Z. Phys.*, **157**, 112 (1959)
[3] M.Y. Thiry, *Compt. Rend. Acad. Sci. Paris*, **226**, 216 (1948).
[4] Jackson, J.D., *Classical Electrodynamics*, 2nd edition, New York: John Wiley & Sons (1975)
[5] Weinberg, S., *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, New York: John Wiley & Sons (1972)
[6] Morris, M., et al., *Phys. Rev. Let.*, **61**, 1446 (1988)
[7] M. Alcubierre, *Class. Quantum Grav.*, **11**, L73 (1994)
[8] Davis, E.W., *Frontiers of Propulsion Science*, in *Progress in Astronautics and Aeronautics*, Am. Inst. of Aeronautics and Astronautics, **227**, 471, (2009)
[9] Overduin, J.M., & P.S. Wesson, *arXiv:gr-qc* 9805018 v1, (1998)
[10] Williams, L.L., in *Space Propulsion and Energy Sciences International Forum*, American Institute of Physics Conference Proceedings, **1103**, 243 (2009).
[11] Millis, M.G., & E.W. Davis, *Frontiers of Propulsion Science*, in *Progress in Astronautics and Aeronautics*, Am. Inst. of Aeronautics and Astronautics, **227**, (2009)
[12] Frieman, J.A., et al., *Ann. Rev. Astron. Astroph.*, **46**, 385 (2008)