The Nature of the Fifth Dimension in Classical Relativity

toward a viable theory of breakthrough propulsion

L.L. Williams

Konfluence Research

SPESIF

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The Barrier to Interstellar Travel

The hyperbolic nature of 4D spacetime constrains the motion of accelerating objects

$$-a^2 = c^2\left(\frac{d^2 t}{d\tau^2}\right)^2 - \left(\frac{d^2 \boldsymbol{x}}{d\tau^2}\right)^2$$

$$c^2 = c^2\left(\frac{dt}{d\tau}\right)^2 - \left(\frac{d\boldsymbol{x}}{d\tau}\right)^2$$

1. you can't go fast enough.

Traveling at the speed of light would require 3 years to the nearest star, and 100,000 years to cross the galaxy (as measured in the rest frame of the galaxy). No object can be accelerated beyond the speed of light.

$$\frac{dx}{dt} = \frac{at}{\sqrt{1 + a^2 t^2 / c^2}}$$

2. you can't go home

Time dilation effects accrue which isolate the traveler temporally from the home planet. While a traveler accelerated at 1 g for 5 years, 74 years would pass on the home planet.

$$t_{rest} = \frac{c}{a}\sinh\left(\frac{a t_{trav}}{c}\right)$$

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Constraints on a Breakthrough Propulsion Theory

- must address general relativity and the 4D geometry of spacetime because this is where the barrier lies
- must address electrodynamics because our technology is ultimately rooted in this force (not the strong or weak forces)
- is expected to be classical, since we are classical beings *(and we have not had much luck with quantum gravity anyway)*
- provides falsifiable predictions which subsume, not abrogate, GR and EM

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The Kaluza (1921) 5D Theory

- fits the constraints of a breakthrough propulsion theory
  - *unites general relativity and electrodynamics*
- simply, general relativity in 5 dimensions
  - *5D Einstein equations provide the field equations*
  - *5D geodesic equation provides the equations of motion*
- new features:
  - *a scalar force field*
  - *a fifth classical dimension*

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Historical Context

1915: publication of general relativity

1921: publication of 5D general relativity

1925: quantum revolution

unified field theories expected to be quantum

1926: 5D theory is quantized and compactified

1948: 5D classical field equations written

2008: scalar field shown to be cosmological

constant scalar field does not constrain terrestrial EM fields

20??: quantum gravity

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5D Metric

(Kaluza 1921)

$$\tilde{g}_{ab} \sim \begin{pmatrix} g_{\mu\nu} & \frac{G^{1/2}}{c^2} A_\mu \\ \frac{G^{1/2}}{c^2} A_\nu & \phi^2 \end{pmatrix}$$

G = gravitational constant

5D proper distance

$$\tilde{g}_{ab} dx^a dx^b \equiv ds^2$$

4D proper time

$$g_{\mu\nu} dx^\mu dx^\nu \equiv c^2 d\tau^2$$

roman indices range over 5 values

greek indices range over 4 values

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Constraining the 5D Theory

- even with just one extra dimension, the resulting equations have far more degrees of freedom than is necessary to unify 4D physics
- the simplest 5D theory assumes (motivated by observation) that none of the fields depend on the fifth dimension – *the cylinder condition*

$$\frac{\partial \tilde{g}_{ab}}{\partial x^5} = 0$$

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The 5D Field Equations

(Thiry 1948)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} \phi^2 T^{EM}_{\mu\nu} + T^{\phi}_{\mu\nu}$$

scalar-tensor (Brans-Dicke) gravity, but with EM source

$$\nabla^{\mu} F_{\mu\nu} \sim F_{\mu\nu} \partial^{\mu} \ln \phi$$

$$F_{\mu\nu} \equiv \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$$

4D EM with but scalar field source

$$\nabla^{\mu} \nabla_{\mu} \phi = \frac{4\pi G}{c^4} \phi^3 F_{\alpha\beta} F^{\alpha\beta}$$

scalar field with EM source

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The Equations of Motion

$$\frac{dU^{\nu}}{d\tau}+\frac{q}{mc}F^{\nu}_{\mu}U^{\mu}+\Gamma^{\nu}_{\alpha\beta}U^{\alpha}U^{\beta}\sim 0$$

standard 4D equations with EM

$$U^{\nu}\equiv\frac{dx^{\nu}}{d\tau}$$

$$\frac{16\pi G}{c^{3}}mA_{\nu}U^{\nu}+q=constant$$

new equation for variation of electric charge

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Energy-Momentum-Charge 5-vector

$$\frac{cdt}{d\tau} = c\gamma \propto energy$$

motion in time

$$\frac{d\mathbf{x}}{d\tau} = \mathbf{v}\gamma \propto momentum$$

motion in space

$$\frac{dx^5}{d\tau} \sim c\frac{q/m}{G^{1/2}} \propto electric\ charge$$

motion in the 5th dimension

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The Nature of the Fifth Dimension

- electric charge is the 4D expression of motion in the fifth dimension
- the fifth dimension is macroscopic
- the fifth dimension is “flat” -- uncurved
  - but can be curved by strong EM fields
- particles with $q/m > G$ are tachyons in the fifth dimension

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Varying Gravitational Constant

- an aspect of standard scalar-tensor (Brans-Dicke) gravity
- arises because *G* is always multiplied by the scalar field
- **5D prediction**: *G* depends on the EM energy density
  - paper explores modified Friedmann equation in a radiation-dominated universe
- field equation for G requires a cosmological scale of variation
  - variations in *G* on a scale of km require 10^{38} erg cm^{-3}, or magnetic fields of 10^{20} G
  - the observed CMBR of 0.25 eV cm^{-3} provides a scale similar to the radius of the universe

**experimentalist note*: *G* is known to only 4 significant figures

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Variation of electric charge

- the equation of motion for the fifth component of the 5D proper velocity shows how electric charge varies with motion in an EM field
- **5D prediction**: electric charge is not a Lorentz scalar
- is this effect undetectable?
  - a relativistic proton with proper velocity *c*, moving in a 10^6 G field with a kilometer lengthscale, would show a variation in charge of only 10^{-31}

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Role of the Gravitational constant

- establishes a cosmic charge-to-mass ratio
- characterizes coupling between gravity and EM
- a geometric scale for the fifth dimension, as the speed of light is for time

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The Barrier Revisited

$$\tilde{g}_{ab}\frac{dx^a}{ds}\frac{dx^b}{ds} = 1$$

$$\frac{d}{ds}\left(\tilde{g}_{ab}\frac{dx^a}{ds}\frac{d\,x^b}{ds}\right) = 0$$

$$\tilde{g}_{\mu\nu} = \eta_{\mu\nu} - \frac{G}{c^4}A_\mu A_\nu$$

$$\tilde{g}_{5\nu} = -\frac{G^{1/2}}{c^2}A_\nu$$

$$\tilde{g}_{55} = -1$$

flat 5D space

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