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Physics of the Electromagnetic Control of Spacetime and Gravity

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Colonization of the stars is a virtual impossibility for our civilization, owing to the nature of space and time described by the theory of relativity. To overcome this barrier will require a breakthrough in physics beyond the current content of relativity theory. Research in fundamental physics for the past century has been focused on the quantum. For the purposes of practical interstellar travel, however, a classical extension to relativity is also of interest. Here we consider classical extensions in particular which unify general relativity and electrodynamics, providing for the electromagnetic control of spacetime and gravity. Such extensions have attractive qualities for the purposes of interstellar travel: a mechanism for electromagnetic distortion of spacetime through terrestrial engineering of a mediating scalar field, and a hyperspace dimension. The theory has no conflict with experiment, but the nature of charge in the theory may provide a testable hypothesis.

Nomenclature

A^μ	electromagnetic potential (4 numbers)
c	speed of light
∇_α	4D covariant derivative
$\widetilde{\nabla}_a$	5D covariant derivative
$F_{\mu\nu}$	electromagnetic force field (6 numbers)
$g_{\mu\nu}$	gravitational potential and the 4D spacetime metric (10 numbers)
$\widetilde{g}_{\mu\nu}$	5D metric (15 numbers)
$\Gamma^\mu_{\alpha\beta}$	gravitational force field (40 numbers); affine connection; Christoffel symbol
$\widetilde{\Gamma}^c_{ab}$	5D affine connection (75 numbers)
γ	Lorentz factor $(1 - v^2/c^2)^{-1/2}$
G	gravitational constant
J^μ	electric charge and electric current (4 numbers)
Λ	cosmological constant
m	mass of a material object
∂_α	4D partial derivative
q	charge of a material object
$R_{\mu\nu}$	Ricci tensor (10 numbers), a non-linear set of derivatives of $g_{\mu\nu}$; the curvature of spacetime
R	$R_{\mu\nu}g^{\mu\nu}$
U^μ	4-velocity of an object moving in spacetime (4 numbers)
$\widetilde{U}^a$	5-velocity of an object moving in 5 dimensions (5 numbers)
s	5D invariant interval
τ	4D invariant interval; proper time
$T_{\mu\nu}$	energy and momentum fluxes of matter and/or radiation (10 numbers)
x^μ	4D spacetime coordinate (4 numbers)
x^a	5D coordinate (5 numbers)
Ω	hypothetical extension of the spacetime interval
ϕ	scalar field of the 5D theory
Ψ	hypothetical field which controls the coupling of mass-energy to gravity

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$\Theta_{\mu\nu}$ hypothetical new source of spacetime curvature (10 numbers)
Υ^{μ} hypothetical new source of electromagnetic fields (4 numbers)
Ξ^{μ} hypothetical new force field (4 numbers)

I. The Impossibility of Interstellar Exploration

Exploration and colonization of the stars by our civilization appears to be effectively impossible according to the theory of special relativity. For consider a moving object with four-velocity

$$U^{\mu} \equiv \frac{dx^{\mu}}{d\tau} = \frac{dt}{d\tau}\frac{dx^{\mu}}{dt} \equiv \gamma \frac{dx^{\mu}}{dt} = \gamma(c, \mathbf{v}) \equiv (U^0, \mathbf{U}) \tag{1}$$

where t is the time coordinate, x^{μ} are the four spacetime coordinates, c is the speed of light, and τ is the Lorentz-invariant proper time.

An axiom of special relativity is the invariance of the spacetime interval $cd\tau$

$$c^2 d\tau^2 \equiv c^2 dt^2 - (dx^2 + dy^2 + dz^2) \equiv dx^{\mu} dx_{\mu} \ . \tag{2}$$

This directly implies that

$$c^2 = (U^0)^2 - (\mathbf{U})^2 \equiv U^{\mu} U_{\mu} = E^2/(mc)^2 - p^2/m^2 \tag{3}$$

where the last terms are written to explicitly reflect the energy E and momentum p for a body of mass m. Equation (3) is the famous relativistic energy equation for massive particles, showing both a kinetic energy and a rest energy.

For any conceivable ship we send to the stars, it must be accelerated. In the interest of simplifying the mathematics without sacrificing the result, let us consider the case of one-dimensional, constant acceleration, a, irrespective of how this acceleration is engineered: chemical rocket, nuclear explosions, anti-matter, lasers, etc.

For acceleration and motion in the x direction, (3) implies

$$c^2 = c^2 \left(\frac{dt}{d\tau}\right)^2 - \left(\frac{dx}{d\tau}\right)^2 \tag{4}$$

This provides one equation for $t(\tau)$ and $x(\tau)$, but a second is needed to determine the system. Therefore consider $dU^{\mu}/d\tau$.

In the frame of the space ship, $\mathbf{v} = 0$ and $\gamma = 1$, yet the acceleration a is still felt inside. So (1) implies $(dU^{\mu}/d\tau)_{ship} = (0, a, 0, 0)$. From this we can calculate the spacetime length of the acceleration 4-vector in the space ship frame. Since the quantity is Lorentz-invariant, it has this same value in any coordinate frame:

$$\frac{dU^{\mu}}{d\tau}\frac{dU_{\mu}}{d\tau} = -a^2 = c^2 \left(\frac{d^2t}{d\tau^2}\right)^2 - \left(\frac{d^2x}{d\tau^2}\right)^2 \tag{5}$$

The simple equations (4) and (5) capture the essence of the problem. They have the solutions:

$$t(\tau) = \frac{c}{a} \sinh\left(\frac{a\tau}{c}\right) \tag{6}$$

$$x(\tau) = \frac{c^2}{a} \cosh\left(\frac{a\tau}{c}\right) \tag{7}$$

Equations (6) and (7) imply the well-known limiting speed of light for an object subject to unlimited acceleration:

$$\frac{dx}{dt} = \frac{at}{\sqrt{1 + a^2t^2/c^2}} \leq c \tag{8}$$

No matter how large the applied force, no spaceship can be accelerated beyond the speed of light, as measured in the frame of the home planet. This limit precludes practical insterstellar travel for our civilization because the distances between the stars are so vast. Our own galaxy is 100,000 light years in diameter, and the mean distance between stars is about 3 light years. Thus, to cross our galaxy would require 100,000 years on the home planet. The galaxy could not be explored by any civilization which persists for a time less than this. Even exploring just 1% of the galaxy could probably not be done during the lifetime of a civilization.

The equations (4) and (5) also imply the well-known effect of time dilation as expressed in (6). A clock on board the space ship measuring an interval of proper time τ would be extremely time-dilated relative to a clock at rest in the home civilization. During the time a spacecraft accelerated at 1 g for 5 years as measured on board, 74 years would pass on the home planet. Yet in this time the travelers could scarcely reach the nearest stars.

Under constant acceleration, the spaceship speed quickly approaches the speed of light. Equation (8) shows that the spaceship can reach half the speed of light by holding 1 g of acceleration for half a year. Such relativistic speeds would dilate the clocks of space travelers enough for a spaceship to travel an arbitrary distance during the lifetime of its crew, but the civilization which sent them can never be informed of their discoveries because it would evolve out of existence in that time. Travelers moving fast enough to reach the stars become temporally detached from their home civilizations and can never return to them.

So the hyperbolic nature of spacetime (2) as expressed in our current understanding of the principles of special relativity sets two basic limits on practical interstellar travel. One is the limiting speed of light, combined with the vast distances to the stars, will prevent a civilization from exploring any significant fraction of the galaxy before the civilization evolves out of existence. The second basic limit is the effect of time dilation, which will temporally disconnect any astronaut from her civilization even though the same time dilation effect would allow the astronaut to go an arbitrary distance in her lifetime.

The infeasibility of galactic exploration by terrestrial civilization is the hard problem of interstellar travel. This is to distinguish it from the other basic problem of interstellar travel, the fuel problem. In comparision, the fuel problem is merely one of engineering a fuel system to propel a space ship at typical rocket speeds over extended durations. The fuel problem is relevant for interstellar travel projects which would send travelers out on slow, multi-generational journeys of no return.

II. An Electromagnetic Machine for Superluminal Travel

Exploration and colonization requires two-way communication with the home civilization on a timescale significantly shorter than the lifetime of the civilization. If our civilization is to colonize any part of the galaxy, it would require some way to break the light barrier and accomplish faster-than-light travel. By whatever means such a thing is accomplished, it would of course require some sort of machine to implement.

In the broadest terms, nearly all the machines built by humans are electromagnetic in nature. Not only are power generation and telecommunication technologies electromagnetic, but chemistry and metallurgy are ultimately electromagnetic phenomena. Furthermore, these achievements are primarily in the area of classical electromagnetism. Quantum electrodynamics is important in materials science but our machines are ultimately classical. In our search for a machine to surmount the light barrier, let us therefore consider extensions to the laws of classical electromagnetism.

Yet the hard problem of practical interstellar travel is one of space and time, as shown in the preceding section. Our current theory of space and time is general relativity which, because gravity is identified with the curvature of space and time, is also a theory of gravity. Any search for an effect to transcend the light barrier with terrestrial engineering must therefore also examine extensions to general relativity. So let us consider the extensions to general relativity.

Certainly there are other approaches to investigating departures from the known laws of physics; here we wish to consider extensions to the laws of spacetime and electromagnetism which may allow us to surmount the light barrier through the electromagnetic control of gravity and spacetime. We start by reviewing the known laws of gravity and electromagnetism, and the known couplings between them.

III. General Relativity and Electromagnetism

The classical equations of general relativity and electromagnetism each decompose into two independent pieces: the field equations and the equations of motion. The former describe the force fields as a function of material sources, the latter describe the motion of material objects under the influence of the force fields. In the following we quote without derivation standard results, in the tensor notation of Jackson[1] and Weinberg.[2]

The tensor formalism is important because a corollary of relativity is that all laws of physics must have a covariant expression; that is, that they have the same mathematical form referenced to any state of motion. Tensors have the proper transformation properties to insure covariance.

III.A. Equations of Motion

To treat motion in gravitational fields requires a generalization of (2):

$$c^2 d\tau^2 \equiv g_{\mu\nu} dx^\mu dx^\nu \tag{9}$$

where $g_{\mu\nu}$ is identified with the metric describing the covariant interval between spacetime events. It is clear from the commutativity of the coordinate length elements in (9) that $g_{\mu\nu}$ must be symmetric in μ and ν. For the 4 dimensions of spacetime, this implies $g_{\mu\nu}$ has 10 components. Equation (3) still holds in that $g_{\mu\nu}U^\mu U^\nu = c^2$.

In the absence of gravitational fields, $g_{\mu\nu}$ is diagonal and constant, and (9) reduces to (2). This is the basic distinction between special relativity and general relativity, and is also the distinction between flat spacetime and curved spacetime. Furthermore, it is an axiom of general relativity that coordinates can always be chosen locally so that (2) holds.

The equations of motion in a gravitational field obtain from the geodesic hypothesis of general relativity:

$$U^\alpha \nabla_\alpha U^\mu \equiv U^\alpha (\partial_\alpha U^\mu + \Gamma^\mu_{\alpha\beta} U^\beta) = \frac{dU^\mu}{d\tau} + \Gamma^\mu_{\alpha\beta} U^\alpha U^\beta = 0 \tag{10}$$

$\Gamma^\mu_{\alpha\beta}$ is the affine connection, a quantity which is formed from derivatives of the metric tensor $g_{\mu\nu}$. ∇_α is the covariant derivative, and ordinary coordinate partial derivatives are abbreviated $\partial/\partial x^\nu \equiv \partial_\nu$.

The effects of gravity are built into the covariant derivative: $\Gamma^\mu_{\alpha\beta}$ is effectively the gravitational field and $g_{\mu\nu}$ is the gravitational potential. Because $g_{\mu\nu}$ has this dual identity as the gravitational potential (10) and the spacetime metric (9), gravity is identified with the curvature of spacetime. We use the terms spacetime and gravity interchangeably in this article.

The equations of motion for a body of charge q and mass m in an electromagnetic field are given by the Lorentz force law (here in cgs units):

$$m\frac{dU^\mu}{d\tau} = \frac{q}{c} F^{\mu\nu} U_\nu \tag{11}$$

where $F^{\mu\nu}$ is the electromagnetic field strength tensor. It has 6 independent components – the vector components of the electric and magnetic fields.

The equations of motion in combined gravitational and electromagnetic fields are then

$$\frac{dU^\mu}{d\tau} + \Gamma^\mu_{\alpha\beta} U^\alpha U^\beta = \frac{q}{mc} F^{\mu\nu} U_\nu \tag{12}$$

III.B. Field Equations

The Einstein equations provide the field equations for $g_{\mu\nu}$,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu} \tag{13}$$

where $R_{\mu\nu}$ and R are rather complicated functions of the metric tensor, $g_{\mu\nu}$. The stress-energy tensor is $T_{\mu\nu}$, which constitutes a source term to the field equations. The Newtonian gravitational constant is G. The constant multiplying the stress-energy tensor is very small, which expresses the relative weakness of the gravitational force. It's why a planet-sized amount of matter is necessary to get significant gravity. The term in Λ is the cosmological constant, and corresponds to energy in the vacuum.

Unlike the Maxwell equations, which were synthesized from centuries of observations of electric and magnetic effects, the Einstein equations (13) were obtained purely theoretically from considerations of general covariance and matching Newton's law of gravity in an appropriate limit. While the predictions of (13) are confirmed many times over, its construction retains an element of artistry. The LHS of (13) has a clear, turn-the-crank prescription in terms of the metric tensor, but the RHS is ad hoc. Einstein himself called this distinction between the two sides of (13) "marble" and "wood".

A suitable stress-energy tensor can be constructed for any source, e.g., massive particles, fluids, or pure radiation. The stress-energy tensor is always constructed so that its covariant divergence vanishes, $\nabla_\mu T^{\mu\nu} = 0$, and this vanishing divergence represents energy and momentum conservation. This conservation property used in construction of the stress-energy tensor is mirrored by the vanishing of the covariant divergence of the "marble" side: $\nabla_\mu (R^{\mu\nu} - Rg^{\mu\nu}/2) \equiv 0$. These 4 equations are called the Bianchi identities and are satisfied identically for any $g_{\mu\nu}$. Naively one may expect that (13) are 10 independent equations in the 10 unknowns $g_{\mu\nu}$. But the 4 Bianchi identities reduce this number to 6 independent equations. The other 4 equations necessary to specify $g_{\mu\nu}$ come from the choice of spacetime coordinates.

The Einstein equations (13) are not linear in $g_{\mu\nu}$, and therefore do not allow any analytic prescriptions for calculating general solutions. However, many exact solutions to the Einstein equations have been discovered, such as the Schwarzschild solution, the relativistic generalization of Newton's law of gravity that describes black holes.

In the specific case where the stress-energy is electromagnetic in origin, the Einstein equations (13) can be solved under conditions of an electromagnetic stress-energy tensor.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T^{EM}_{\mu\nu} \tag{14}$$

where the electromagnetic stress-energy tensor is

$$T^{\alpha\beta}_{EM} = \frac{1}{4\pi} \left(g^{\alpha\mu} F_{\mu\lambda} F^{\lambda\beta} + \frac{1}{4} g^{\alpha\beta} F_{\mu\lambda} F^{\mu\lambda} \right) \tag{15}$$

Now let us consider the field equations for electromagnetism, the Maxwell equations, first in the limiting case of flat spacetime (no gravity), in their covariant form (here in cgs units):

$$\partial_\nu F^{\nu\mu} = \frac{4\pi}{c} J^\mu \tag{16}$$

where $F^{\nu\mu} \equiv \partial^\nu A^\mu - \partial^\mu A^\nu$.

The form of (16) manifests the Lorentz invariance of the Maxwell equations. The electromagnetic current 4-vector $J^\mu \equiv (\rho c, \mathbf{J})$ comprises the 3 spatial components of electric current, and the one temporal component of electric charge density. The electromagnetic potential 4-vector $A^\mu \equiv (\Phi, \mathbf{A})$ comprises the electric potential and the magnetic vector potential.

At first glance, (16) would appear to be 4 equations in the 4 unknowns A^μ. However, conservation of charge requires that $\partial_\mu J^\mu = 0$, which constrains the 4 equations. This constraint reduces the number of independent equations to 3. The extra degree of freedom is fixed by the choice of a gauge – this is the famous gauge invariance of electromagnetism. It is analagous to the coordinate invariance of general relativity described above.

Because the gravitational potential $g_{\mu\nu}$ is so intimately tied to the properties of spacetime, the Principle of General Covariance which underpins general relativity provides a simple prescription for writing the field equations of electromagnetism in the presence of gravitational fields. They are obtained from the flat-spacetime Maxwell equations (16) by replacing the partial derivative ∂_μ with a covariant derivative ∇_μ:

$$\nabla_\nu F^{\nu\mu} \equiv \partial_\nu F^{\nu\mu} + \Gamma^\nu_{\nu\alpha} F^{\alpha\mu} + \Gamma^\mu_{\nu\alpha} F^{\nu\alpha} = \frac{4\pi}{c} J^\mu \tag{17}$$

The terms multiplying $\Gamma^\nu_{\alpha\beta}$ and $F_{\alpha\beta}$ represent the effects of gravity on electromagnetic fields.

III.C. Limits of Wormholes and Space Warps

Some solutions of the Einstein equations which have implications for interstellar travel have been found, solutions which offer the promise of surmounting the light barrier. The Einstein equations (13) or (14) allow for distortions in spacetime, wormholes, which can connect spatially disparate parts of the galaxy and which therefore could in principle be traversed in arbitrarily short times.[3] (Such wormholes need not be generated electromagnetically). Wormholes are extreme deformations of spacetime, similar in some ways to black holes, at different locations in the galaxy. They could feasibly be "connected" to each other. A traveler would descend into one wormhole and pop out in the other a short time later but very far away. So curved spacetime offers a way to accomplish faster-than-light travel in principle. Such superluminal travel is still prohibited locally, but spacetime itself can be curved to connect different points in space.

We already have an engineering problem because even if a civilization had the wherewithal to engineer wormholes, the civilization would presumably still have to travel sub-luminally across the galaxy to the "exit" point and build it.

There is another approach to interstellar travel which leverages curved spacetime. Alcubierre[4] discovered a "warp drive" solution to the Einstein equations in which a specially engineered spacetime bubble can transport its occupants superluminally across the galaxy. Like the wormhole solutions, spacetime only allows locally sub-luminal travel, but in this case general relativity allows a bubble of flat space to move superluminally. This at least is better than the wormhole approach because one need not cross the galaxy first sub-luminally to build the transport system.

Unfortunately, both the wormhole and the Alcubierre solutions would require the engineering of astronomical amounts of mass-energy, and even exotic *negative energy*, to warp spacetime into the desired configurations of 100-meter-size wormhole throats or warp bubbles,[5].[6] Negative energy provides a gravitational repulsion, and its science is tenuous. Yet even if negative energy could be manufactured in macroscopic amounts, the root of the impractibility is the very small number G/c^4 which provides the coupling between energy and spacetime curvature in (13) and (14).

A major objection to faster-than-light travel from special relativity is that such travel could lead to breakdowns in causality; that an effect can precede its cause. This is an extra assumption not strictly built into the theory of relativity, albeit one with strong support in the scientific community. The wormhole and Alcubierre solutions give us some hope that in spite of causality concerns, faster-than-light travel is allowed in nature.

So, while feasible within the context of our understanding of physical law, wormholes and Alcubierre drives are impractical for the solar-mass or Jupiter-mass engineering and exotic energies which would be required to realize them. These solutions to the Einstein equations give us hope that faster-than-light travel is not impossible in principle, but they have no practical realization yet for terrestrial engineering. In order to make interstellar travel feasible for a civilization existing for a limited time around some star, qualitatively new extensions to the known laws of physics will be required.

IV. New Couplings Between Electromagnetism and Gravity

To postulate new laws of physics, to search for new physical effects, can be perilous. In our search for possible new couplings between spacetime/gravity and electromagnetism, we are guided by the principle of general covariance: that any equation of physical law must preserve its form under a general coordinate transformation. An expression of the fundamental nature of spacetime, irrespective of the forces and fields, this principle has been applied with great success across all fields of physical law since its introduction by Einstein over a century ago. While there is no "relativity force" per se, as there is an electric force, for example, the application of the principle of general covariance has yielded our understanding of gravity as a coordinate-dependent phenomenon. We therefore continue to enforce general covariance as a discriminator for any new theory.

Yet, as described above, it is considerations of special relativity and Lorentz invariance (2) which set the limits on practical interstellar travel that we wish to surmount. So we allow some relaxation of the Lorentz coordinate transformation per se, but still demand that the laws of physics retain their mathematical form under the more general coordinate transformation we wish to contemplate. Note that in general relativity, general covariance and Lorentz invariance are independent conditions.

As we consider these extensions to physical law, we keep in mind that we are not overthrowing the Maxwell or Einstein equations, but instead we are seeking a new regime of operation which has not been experimentally accessed before. For example, the original equations synthesized by Maxwell (16) are now understood to include extra general-relativistic effects described in (17); (16) is understood to be the limiting case of (17). Yet these effects were not seen by Maxwell because they are negligible on earth's surface where Maxwell's equations were first discovered.

In this spirit we contemplate a relaxation of invariance under the Lorentz transformation, and with it the limiting speed of light. We saw how the fundamental limit to interstellar travel arises from (2). To surmount the light barrier would presumably require an extension to (2) and its generalization (9):

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = c^2 d\tau^2 \pm d\Omega^2 \tag{18}$$

where Ω is some yet-to-be-discovered function of the coordinates x^{μ}. This function is suggestively written as a differential to accord with the general form of (9); it characterizes the deviation from true Lorentz invariance. Multiplicative corrections to (9) are dispensed with because the terms in $dx^\mu dx^\nu$ would be indistinguishable from an arbitrary metric. The extension (18) to (9) is therefore presumed to be additive.

Of course, the search for Lorentz violations has been under way for a century through a variety of different experiments. No violation has been seen yet and there is no evidence for Lorentz violations in the environments tested by the various experiments. So if they exist at all, Lorentz violations must be in a test regime that so far has not been explored – for example, a precisely engineered variation of known or unknown force fields, but a variation that does not occur under natural conditions. An example of emergent forces of this type is the vacuum fluctuation force of the Casimir effect; the Casimir force is not felt by natural macroscopic systems unless a machine is built which expresses it.

Another generally-covariant extension to physical law with relevance to interstellar travel is an extension to (13) which would obviate the need for astronomical amounts of mass/energy to warp spacetime into wormholes or Alcubierre drives. Let us therefore consider some new control of the coupling constant G/c^4 multiplying the stress-energy tensor:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} \Psi T_{\mu\nu} + \Theta_{\mu\nu} \tag{19}$$

where Ψ is a function of the coordinates x^μ. The function $\Theta_{\mu\nu}$ constitutes some yet-to-be-discovered source of spacetime curvature. The cosmological constant term is dropped from (19) but it could be considered implicit in $\Theta_{\mu\nu}$. Two new fields, Ψ and $\Theta_{\mu\nu}$, are introduced to the Einstein equations; they are presumed to be jointly constrained by the Bianchi identities.

For extensions to the Maxwell equations, there is no need to contemplate adjustments to the coupling constant on the RHS of the Maxwell equations (17) as we did with Ψ in (19) – the electromagnetic forces are already quite strong and easy to generate through terrestrial engineering. But we do allow another source of electromagnetic fields Υ^μ:

$$\nabla_\nu F^{\nu\mu} = \frac{4\pi}{c} J^\mu + \Upsilon^\mu \tag{20}$$

Finally, additional forces Ξ^μ are contemplated in the equations of motion (12):

$$\frac{dU^\mu}{d\tau} + \Gamma^\mu_{\alpha\beta} U^\alpha U^\beta = \frac{q}{mc} F^{\mu\nu} U_\nu + \Xi^\mu \tag{21}$$

Although they have been introduced separately, the quantities Ω, Ψ, $\Theta_{\mu\nu}$, Υ^μ, and Ξ^μ, are expected to be related by new, covariant field equations, as well as by the constraints of charge conservation and the Bianchi identities. So the number of degrees of freedom are going to be smaller than one might naively expect from independently adding new terms to the field equations and equations of motion. Indeed, as we will see when we consider a specific theory in the next section, a single yet-to-be-discovered scalar field is sufficient to account for all these effects.

Of the generalized extensions (18), (19), (20), (21), perhaps (19) has been the most studied. The classic Brans-Dicke extensions of general relativity as described by Weinberg[2] are of the form (19). Brans-Dicke theory posits a scalar field Ψ in addition to the standard metric $g_{\mu\nu}$; the Brans-Dicke scalar field equation has matter as its source. The stress tensor $\Theta_{\mu\nu}$ is a function of Ψ, but Brans-Dicke theory assumes the scalar field does not enter the equations of motion: $\Xi^\mu = 0$. Brans-Dicke makes no reference to the Maxwell equations.

More recent considerations of (19) stem from late-20th-century discoveries in cosmology. Specifically, the acceleration of the Hubble expansion is modeled mathematically as a cosmological constant in (13). Considerations of quantum theory also suggest that there is a vacuum energy which would manifest as a cosmological constant. And the standard model of cosmology has an early era of rapid inflation which is modeled mathematically with a scalar field that manifests in (19) as $\Theta_{\alpha\beta}$.[7]

Although a new scalar field Ψ would not be unheard of, any discovery of a new force Ξ^μ in the equations of motion or a new source of electromagnetic fields Υ^μ would be truly revolutionary. Let us now consider a specific theory which illustrates the sort of extensions considered above.

V. The Example of Five-Dimensional Relativity

Soon after Einstein's completion of general relativity, Kaluza[8] introduced a theory which unified general relativity and classical electromagnetism. This was done by writing the Einstein equations and the geodesic equation in 5 dimensions using the five-dimensional (5D) metric $\widetilde{g}_{ab}$:

$$\widetilde{g}_{\mu\nu} = g_{\mu\nu} - k^2\phi^2 A_\mu A_\nu \quad , \quad \widetilde{g}_{5\nu} = -k\phi^2 A_\nu \quad , \quad \widetilde{g}_{55} = -\phi^2 \quad , \quad k^2 = 16\pi G/c^4 \tag{22}$$

Here greek indices continue to be used to span the 4 dimensions of spacetime, the index 5 signifies the fifth dimension, and roman indices span all 5 dimensions. A tilde is used to denote 5D quantities. Thus, seen as a matrix, the 5D metric $\widetilde{g}_{ab}$ consists of the 4D metric $g_{\mu\nu}$ "framed" by the electromagnetic potential A^μ and a new scalar field ϕ^2 at the 5th diagonal.

This theory has a 5D invariant length element $ds^2 \equiv \widetilde{g}_{ab}dx^a dx^b$. From (22) one immediately obtains the extension similar to (18):

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu - \phi^2(kA_\nu dx^\nu + dx^5)^2 \tag{23}$$

Note that comparison of (22) or (23) with (2) shows that the 5th dimension has a spacelike signature. To reproduce standard 4D physics requires the 5th dimension have the same sign in the metric as the space coordinates.

A key assumption of the 5D theory is *the cylinder condition*, which is that none of the fields depend on the 5th coordinate: $\partial_5\widetilde{g}_{ab} = 0$. The cylinder condition is partially out of convenience: it enormously simplifies the theory, yet still provides some non-trivial predictions not found in 4D physics. Although aesthetically unappealing, it can be viewed simply as a boundary condition on our world – we don't see a fifth dimension. Still, the ad hoc nature of the cylinder condition is perhaps the main weakness of the theory.

The 5D field equations for $g_{\mu\nu}$, A^μ, and ϕ are then obtained from the 5D vacuum Einstein equations

$$\widetilde{R}_{ab} - \frac{1}{2}\widetilde{g}_{ab}\widetilde{R} = 0 \tag{24}$$

applied to (22) under the constraint of the cylinder condition, resulting in a set of extensions to physical law similar to (19) and (20):

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}\phi^2 T^{EM}_{\mu\nu} + T^\phi_{\mu\nu} \tag{25}$$

$$\nabla^\mu F_{\mu\nu} = -3F_{\mu\nu}\partial^\mu \ln\phi \tag{26}$$

$$\nabla_\alpha\nabla^\alpha\phi = \frac{4\pi G}{c^4}\phi^3 F_{\alpha\beta}F^{\alpha\beta} \tag{27}$$

$T^\phi_{\mu\nu}$ is a stress-energy tensor for the scalar field whose precise functional dependence on ϕ will not concern us here. The field equations were first obtained by Thiry[9] and by Jordan & colleagues;[10] see Bargmann[11] for an early review. The vacuum 5D theory provides the Einstein equations for $g_{\alpha\beta}$, the Maxwell equations for A^α, and a field equation for ϕ – an elegant unification of the Maxwell and Einstein equations.

Kaluza originally assumed ϕ was constant. In this limit, (25) reduces to (14) and (26) reduces to the source-free limit of (17). However, this assumption restricts the electromagnetic field through (27). Only decades later did Jordan and Thiry independently obtain the full set of self-consistent field equations (25), (26), and (27).

If the cylinder condition is relaxed, an enormous number of degrees of freedom are made available, and it's not at all clear how to interpret the additional terms comprising $\Theta_{\mu\nu}$ in (19). Overduin & Wesson[12] suggest an interpretation of the x^5 derivatives in terms of material sources such as $T_{\mu\nu}$ and J^μ. In this way, matter and forces are both obtained purely from geometry. While such possibilities cannot be ruled out, the extra complexity is not necessary for the purposes of this article.

The 4D equations of motion in this theory are obtained from a 5D geodesic equation:

$$\widetilde{U}^b\widetilde{\nabla}_b\widetilde{U}^\mu = \frac{d\widetilde{U}^\mu}{ds} + \widetilde{\Gamma}^\mu_{ab}\widetilde{U}^a\widetilde{U}^b = 0 \tag{28}$$

where $\widetilde{U}^a \equiv dx^a/ds$. Now use (22) to recast (28) as

$$\frac{d\widetilde{U}^\mu}{ds} + \Gamma^\mu_{\alpha\beta}\widetilde{U}^\alpha\widetilde{U}^\beta = k\phi^2\widetilde{Q}g^{\mu\beta}F_{\beta\alpha}\widetilde{U}^\alpha - \frac{\widetilde{Q}^2}{2}\partial_\alpha\phi^2 \tag{29}$$

where $\widetilde{Q} \equiv -(\widetilde{U}^5 + k\widetilde{A}_\alpha\widetilde{U}^\alpha)$. Equation (29) has been studied in various forms by many authors.[13–16]

To make contact with standard theory, cast (28) in terms of $U^a \equiv dx^a/d\tau$:

$$\frac{dU^\mu}{d\tau} + \widetilde{\Gamma}^\mu_{\alpha\beta} U^\alpha U^\beta + 2\widetilde{\Gamma}^\mu_{5\alpha} U^\alpha U^5 + \widetilde{\Gamma}^\mu_{55} (U^5)^2 + U^\mu \frac{d}{d\tau} \ln\left(\frac{cd\tau}{ds}\right) = 0 \tag{30}$$

where the $\widetilde{\Gamma}^a_{bc}$ are constructed as in 4D from derivatives of $\widetilde{g}_{ab}$.

Agreement with the standard equations of motion (12) requires the identification

$$kU^5 \equiv k\frac{dx^5}{d\tau} = \frac{q}{mc} \tag{31}$$

In other words, electric charge arises from "motion" in the 5th dimension. With this identification and (30) we can write the extensions Ξ^μ to the equations of motion introduced in (21)

$$\Xi^\mu_{\rm 5D} = -\frac{q}{mc} U^\alpha A_\alpha \, \partial^\mu \phi^2 - \frac{(q/m)^2}{32\pi G} \partial^\mu \phi^2 + U^\mu \frac{d}{d\tau} \ln\left(\frac{cd\tau}{ds}\right) + O(k^2 A^2) \tag{32}$$

This equation is correct to first order in the small dimensionless quantity $kA << 1$. Because k as given by (22) is so small, even an astronomical electromagnetic field such as the 10^{12} Gauss magnetic field of a neutron star would still have $kA \sim 10^{-6}$. When the scalar field is constant, (32) reduces to only the second-order terms.

VI. Implications of the 5D Theory

A key aspect of the theory with respect to interstellar travel is the modified coupling constant between energy and spacetime curvature (25). The scalar field ϕ acts to tune the coupling between electromagnetic energy and spacetime curvature. Control of the scalar field would therefore seem to offer control of gravity.

The theory predicts that variation in the scalar field arises from electromagnetic sources; the field equation (27) provides a lengthscale for the variation of ϕ. For a neutron star magnetic field of 10^{12} Gauss, the lengthscale is of order one astronomical unit.[17] Of course, the variation induced in ϕ from terrestrially-engineered electromagnetic fields will be negligible – the scale of variation would be the size of the universe. Just as the small coupling constant G/c^4 in (14) requires astronomical amounts of mass-energy to curve spacetime, the same small coupling constant in (27) requires astronomical amounts of electromagnetic energy to "put a dent" in the scalar field. This may be troubling for our hope to use electromagnetic means to engineer the scalar field. However, in spite of its long history a key element remains missing from the theory – source terms in (24). Some work has been done in this area by Nodvik.[18] But without an understanding of material sources of the scalar field, the theory remains incomplete. Nevertheless we have here a framework for the electromagnetic control of gravity: mediated through a scalar field which couples to both electromagnetism and gravity.

The prospect of a universal scalar field is taken seriously in light of the developments in cosmology over the past quarter century. A period of inflation early after the Big Bang is modeled with a scalar field. The dark energy driving the excess galactic expansion accounts for 70% of the energy content of the universe. Although this quantity is observationally well-modeled by a cosmological constant,[7] it is indeed a scalar field. Finally, there is the long-standing evidence for dark matter. This, too, can be described in terms of a scalar field. Inflation, dark energy, and dark matter are empirical constructs that are not well-integrated into the rest of physics. The 5D theory offers a framework to do so.

Another attractive feature of the 5D theory from the standpoint of interstellar travel is the existence of the fifth dimension – a hyperspace. We know that the fifth dimension must have a spacelike signature in the metric. Furthermore, motion in the fifth dimension manifests as electrical charge in 4D. This is suggestive that the theory harbors some mechanism whereby manipulation of electric charge can be used to shortcut paths in 4D spacetime with motion in the fifth dimension. This is especially promising because the "speeds" in the fifth dimension can be superluminal: of order $10^{20}c$ for electrons,[17] e.g.

Although not beneficial for superluminal travel, the theory offers a prospect for additional propulsive forces based on coupling with the scalar field. Note from (31) and (22) that the gravitational constant sets a universal charge-to-mass ratio: it has units of q^2/m^2. Sub-atomic particles such as electrons and protons have large charge-to-mass ratios when expressed in units of $G^{1/2}$. For such particles, the second term on the RHS of (32) could be quite large; the manifestation of these forces is controlled by the variation of the scalar field. Furthermore, such large charge-to-mass ratios imply spacelike 5D intervals in (23).

VII. Detecting a Variation in Charge

The 5D theory as described in (25), (26), (27), and (30) is entirely consistent with known physical law; no contradictions with known physics have been discovered. It would seem that the 5D hypothesis presents an alternative way of ordering the combined laws of gravity and electromagnetism, plus the physics of a scalar field. Because of the unsatisfying nature of invoking the fifth dimension only to throw it away with the cylinder condition, the 5D theory has not garnered mainstream acceptance. Some new prediction is required to validate the theory. The variation in electric charge may answer this requirement.

A unique prediction of the 5D theory is that electric charge is not a true Lorentz scalar; it depends on particle 4-velocity and on ambient electromagnetic fields. Electric charge is the fifth component of an energy-momentum-charge 5-vector. But it turns out the cylinder condition requires electric charge behave approximately like a Lorentz scalar.

To see this, start with the 5D equations of motion (30) applied to (22):

$$\frac{dU^\nu}{d\tau} + \Gamma^\nu_{\alpha\beta} U^\alpha U^\beta = k\phi^2 Q g^{\nu\beta} F_{\beta\alpha} U^\alpha + \frac{Q^2}{2} g^{\nu\alpha}\partial_\alpha \phi^2 - U^\nu \frac{d}{d\tau} \ln\left(\frac{cd\tau}{ds}\right) = 0 \tag{33}$$

where $Q \equiv U^5 + kA_\nu U^\nu$. Applying the cylinder condition to the 5D geodesic equation results in a conserved quantity along 5D paths:

$$\partial_5 \widetilde{g}_{ab} = 0 \Longrightarrow \widetilde{g}_{5b}\widetilde{U}^b = -\phi^2(kA_\nu \widetilde{U}^\nu + \widetilde{U}^5) = -\phi^2 \frac{d\tau}{ds} Q = -\phi^2 \widetilde{Q} = \text{constant} \tag{34}$$

From (22) it follows that

$$\left(\frac{cd\tau}{ds}\right)^2 = 1 + \widetilde{Q}^2\phi^2 \tag{35}$$

which, combined with (34), implies the quantity $d\tau/ds$ depends only on the scalar field.

In the 5D flat-space limit that the scalar field is constant and the 4D metric is Minkowskian, $\eta_{\alpha\beta}$, the equations of motion reduce to

$$\frac{dU^\nu}{d\tau} = kQ\eta^{\nu\beta} F_{\beta\alpha} U^\alpha = \left(\frac{q}{mc} + \frac{16\pi G}{c^4} A_\mu U^\mu\right) F^\nu{}_\alpha U^\alpha \tag{36}$$

where Q is constant. Even in the 5D-flat limit of the cylinder condition and constant scalar field, modifications to the Lorentz force persist.

The quantity $\phi^2\widetilde{Q}$ is conserved for a charge in motion, but in the absence of electromagnetic fields and with constant ϕ, the charge q is invariant. Thus the cylinder condition accounts for why charge behaves as a Lorentz scalar. With electromagnetic fields, however, there can be minute variations in charge. Because $kA \ll 1$, the variation in charge, if it exists, could have gone undetected. An observation of non-Lorentzian charge variation would validate the picture of 5D relativity.

VIII. Conclusions

Exploration and colonization of all but a very few nearby stars in the galaxy is impossible owing to the limiting speed of light and the nature of time dilation. Any machine which allows us to surmount the light barrier must somehow manipulate these constraints. Furthermore, wormholes and Alcubierre warp drives, while ostensibly surmounting the light barrier, are not achievable with terrestrial engineering. Given the facility of our civilization with electromagnetic forces, a consideration of extensions to physical law which unite general relativity and electromagnetism is therefore of interest. The theory of 5D general relativity provides just such a framework.

In 5D general relativity, electromagnetism and gravity are unified through a scalar field which is manipulated by electromagnetic means. The scalar field controls the coupling of electromagnetic stress-energy to spacetime curvature, thus presenting the prospect for realizing wormholes or Alcubierre warps with terrestrial amounts of energy. The scalar field could be identified with one of the mysterious scalar fields discovered cosmologically: inflation, dark matter, and dark energy.

The theory also contains a hyperspace dimension, and "motion" in this dimension manifests as electric charge in spacetime. This suggests a prospect for shortcuts through spacetime in the fifth dimension by manipulating electric charge.

While the theory is entirely consistent with experiment, it predicts that electric charge is not a true Lorentz scalar but rather depends on particle motion and electromagnetic fields. The variation is minute, but may provide a testable way to verify the theory.

Although 5D relativity has long been known to researchers, it was dropped soon after its discovery a century ago for several reasons: there was no motivation for the scalar field, the cylinder condition seemed unsatisfying, and it was thought a more-fundamental theory of gravity would involve the quantum. Now we know that quantum gravity may well be impossible, that there are scalar fields at work in the cosmos, and that the vacuum state of the universe can be the result of a broken symmetry. So far it appears 5D relativity indeed offers a viable approach to electromagnetic control of spacetime and gravity.

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