Back to the Future: Rise of the Scalar Field and its Implications for Interstellar Travel

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The recent experimental confirmation of the Higgs boson, the last piece of the standard model of particle physics, is also the discovery of the first scalar field in nature. Even as we celebrate the triumph of the quantum scalar field, we are puzzled by the mysterious scalar fields operating on the scale of galaxies. At this turning point in physics, let us assess the role of scalar fields in physical law; assess what we know about scalar fields operating in our universe; assess their implications for interstellar travel. We look back to the lessons of physics before the quantum revolution, lessons lost from physics at a time when scalar fields seemed implausible and quantum gravity seemed imminent. We discover that scalar fields offer the promise of the electromagnetic control of spacetime and gravity, and we consider to what extent the already-discovered scalar fields can be understood in those terms.

1. INTRODUCTION

Remarkable discoveries in cosmology made over the past couple of decades have revealed that our universe is dominated by mysterious force fields operating on galactic scales. These mysterious force fields are parameterized as scalar fields in the equations of physics – no one knows what they really are. This article surveys the modern history of scalar fields in physics, culminating with their proliferation in cosmology at the end of the 20th century. Our lens for examining and describing the scalar fields is the Friedmann equation of cosmology. The article describes how these recent discoveries argue for a unified field theory encompassing general relativity and electrodynamics. Such a unification could have profound implications for some key aspects of the problem of interstellar travel: the time-distance problem, gravity control, and a hyperspace dimension.

2. FORCE FIELD TAXONOMY

The designation "scalar field" refers to a particular type of force field. The designation "field" is the general term for the mathematical description of force. The electric force, for example, arises as a result of interaction with the electric field. We now understand our universe to be much in the way of waves scattering and shimmering on several overlapping oceans of force – oceans of force we call "fields".

Mathematically, there are several types of fields. They differ in how they change mathematically under a coordinate transformation. They also differ in the number of field values at each point in space. A really complex field can have many different numbers characterizing it at each point in space.

A *scalar field*, then, is a field that does not change at all under a coordinate transformation. It is characterized by a single number, e.g. ϕ, at each point in space. That single number can be any function of the coordinates. The Newtonian gravitational field is a scalar field, and most of what we know as the gravitational force is merely due to gradients in this universal scalar field.

A *vector field* is a field that transforms like a vector under a coordinate transformation. Vector fields are characterized by 4 numbers at each point in space, e.g., A^μ. Here, the greek letter μ is understood to take on any of 4 values. The electromagnetic field is a vector field. The 6 components of the electric and magnetic 3-component vectors are gradients in the electromagnetic 4-component vector (4-vector).

A *tensor field* is a field that transforms like a tensor under a coordinate transformation. The most common tensors are rank 2, e.g., $g_{\mu\nu}$. As with vector fields, each index can range over 4 values, so a tensor field can have as many as 16 values at each point in space – that's really complex! If the tensor is symmetric, then the number is 10. The gravitational field as described by general relativity is a symmetric rank-2 tensor field, characterized by 10 numbers at each point in space.

3. NEWTONIAN FRIEDMANN EQUATION

Our survey of scalar fields in modern physics shall be through the lens of the Friedmann equation of cosmology, which is used to describe the evolution of the universe from the Big Bang onward. Although the Friedmann equation is properly derived from general relativity, in hindsight we see we can obtain it with simple Newtonian physics. Understanding the Newtonian picture gives insight into the relativistic Friedmann equation.

Consider a sphere of mass M and radius $R(t)$ that is changing with time. The acceleration of any element of mass Δm at R will be described by Newton's laws:

$$\Delta m \frac{d^2R}{dt^2} = -\frac{GM\Delta m}{R^2} \tag{1}$$

Multiply each side by dR/dt and integrate to find:

$$\frac{1}{2}\left(\frac{dR}{dt}\right)^2 = \frac{GM}{R} + U_0 \tag{2}$$

where U_0 is an integration constant.

Reparameterize $M = 4\pi\rho R^3/3$ in terms of a mass density ρ, and $R = a(t)r$ in terms of a unitless, time-dependent scale factor $a(t)$. Then find:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho + \frac{2U_0}{r^2a^2} \tag{3}$$

Here $\dot{a}$ is a shorthand for da/dt. The Newtonian Friedmann equation, then, relates the change in size of the mass to it's density – simple as that! It contains nothing more than Newton's Second Law and Newton's Law of Gravity, integrated once to describe the energetics.

4. GENERAL RELATIVITY – 1915

We start our survey of modern scalar fields with general relativity, the lynchpin of our description of classical scalar fields, and of our description of cosmology. It is of course the relativistic theory of gravity developed primarily by Einstein [1], but with contributions from others. The gravitational field is described by a symmetric tensor $g_{\mu\nu}$, describing 10 numbers at each point in space and time (spacetime). This tensor $g_{\mu\nu}$ that describes the gravitational field is known as the spacetime *metric*. The field equations for $g_{\mu\nu}$ are the Einstein equations:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu} \tag{4}$$

The indices mean that the equation written here is a compact form of 10 separate equations. The terms on the left side, $R_{\mu\nu}$ and R, are complicated, non-linear functions of derivatives of $g_{\mu\nu}$ that are interpreted as spacetime curvature. The quantity $T_{\mu\nu}$ is the stress-energy tensor, a set of energy and momentum densities that can deform spacetime and create relativistic gravitational fields. The gravitational constant is G, and the speed of light is c.

The Einstein equations are a generalization of Newton's Law of Gravity. Written in terms of a scalar gravitational potential ϕ_N and a mass density ρ, Newton's Law would read:

$$\nabla^2\phi = 4\pi G\rho \tag{5}$$

This is a single, scalar equation in the Newtonian gravitational field ϕ, for which (4) is the relativistic generalization.

It turns out that ϕ_G is related to the time-time component of $g_{\mu\nu}$, one of the 10 components. For studies of cosmology, symmetries are invoked to reduce the number of components of the metric tensor from 10 to a few. In the end, scalar field equations not too disimilar from (5) or (3) are obtained from (4) for the reduced components.

5. FRIEDMANN EQUATION – 1922

With the equations of general relativity in place, Friedmann was able [2] to derive the relativistic generalization of (3). By assuming spherical symmetry, isotropy, and uniformity, the 10 components of $g_{\mu\nu}$ can be reduced to two parameters: a scale factor $a(t)$ and a curvature index κ. These two numbers parameterize a family of spacetime metrics called the Robertson-Walker metric. Applying the Robertson-Walker metric to the 10 equations of (4) yields the single scalar equation attributed to Friedmann:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\mathscr{E}(t) + \cancel{\frac{\kappa c^2}{R_0^2 a^2(t)}} \tag{6}$$

The scale factor $a(t)$ parameterizes the expansion of the universe, just as in equation (3). The time coordinate t is a cosmic time coordinate that goes to zero at the Big Bang. The total energy density of the universe is $\mathscr{E}$. The curvature index κ takes on the values of 0 (flat universe), $+1$ (positive-curvature universe), or -1 (negative curvature universe). It turns out that our universe appears to be the special case of the flat universe. Therefore the curvature term is zeroed out for our universe.

We find, then, that the relativistic Friedmann equation (6) is quite similar to the Newtonian Friedmann equation (3), just with ρ replaced by $\mathscr{E}/c^2$, as might be expected from a relativistic generalization. But instead of the expansion or contraction of a fixed mass, as in the Newtonian derivation, (6) and the scale factor $a(t)$ describe how spacetime itself is expanding. The dynamics of (6) appear mathematically similar to (3) because both are driven by gravity.

Since the work of Hubble, it has been known that the distant galaxies are receding. The measure of this recession is called the Hubble constant, usually given in kilometers-per-second-per-megaparsec. Farther galaxies recede faster. The accepted value of the Hubble constant is about 70 km/(s Mpc). In the equation (6), the Hubble constant $H = \dot{a}/a$. So the left side of (6) is an observable. A large part of the quest of modern cosmology is to understand all the pieces that go into $\mathscr{E}$, the total universal energy density. It includes energy of any sort, from the mass-energy of visible matter to the dark energy of the vacuum. These components will be described below.

6. COSMOLOGICAL CONSTANT – 1917

Not long after creating general relativity, Einstein applied his field equations to the universe as a whole [3], even before the work of Friedmann. At the time, the recession of the galaxies was not known. Therefore Einstein sought a static solution to (4) for the universe, since he thought the universe was static. He realized his equations of 1915, (4), would not support a static solution. So he proposed an extension to them that was still consistent with all the other symmetry properties that (4) was based upon, but which would allow him to obtain a static solution. The additional term Λ was called *the cosmological constant*:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{7}$$

The cosmological constant has the same value everywhere in the universe. We could make the analogous modification of Newton's Law of Gravity (if we had a reason to):

$$\nabla^2 \phi + \Lambda = 4\pi G \rho \tag{8}$$

The modification to the Friedmann equation is:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2} \mathscr{E}(t) + \frac{\Lambda}{3} \tag{9}$$

We can see that the cosmological constant is a particularly simple scalar field: one that is constant everywhere. There can therefore be no forces associated with gradients of this scalar field. But the cosmological constant does behave dynamically, by contributing an effective energy density $\mathscr{E}_\Lambda = c^2 \Lambda / 8\pi G$ to the Friedmann equation that serves to (in part) drive the Hubble expansion. This constant effective energy density means that energy is contained in the vacuum, and is continually created in the vacuum as the universe expands.

Furthermore, if there is only Λ and no other energy density $\mathscr{E}$, then (9) implies the vacuum energy of the cosmological constant drives an exponential expansion of the universe:

$$a(t) \sim \mathrm{e}^{H_\Lambda t} \tag{10}$$

In the end, Hubble's discoveries led Einstein to renounce his ill-fated addition to the field equations. But the concept would be dusted off again decades later, with motivation from the quantum revolution.

7. KALUZA UNIFICATION – 1921, 1948

In 1919, Einstein received a communication from the obscure Polish mathematician Theodor Kaluza, showing how, if general relativity were formulated in 5 dimensions instead of the usual 4, both 4-dimensional general relativity and electrodynamics could be recovered. The framework also required an additional scalar field that had no plausible reflection in the physics of the day. It was a stunning formal unification of gravity and electromagnetism.

Unbelieving, Einstein vacillated for two years between rejection and admiration. Finally, in 1921, Einstein submitted Kaluza's paper [4] to the journals (in those days, only eminent scientists could submit scientific articles to journals, and apprentices submitted their papers to eminent scientists).

Kaluza's hypothesis was remarkable at the time, and led to far-reaching changes in the modern approach to unified field theory. It seems fair to say the results have not been fully absorbed or understood, but they seem to reveal something profound.

Kaluza hypothesized that there is another dimension besides the usual 4 of space and time – and that the equations of general relativity would hold in the 5 dimensions just as they hold in 4. This latter stipulation is not unreasonable; the Einstein equations are derived from profound considerations of coordinate invariance, and those considerations are not really dependent on the number of dimensions per se. The curvature tensors only get more interesting in higher dimensions.

The Kaluza hypothesis at its essence is to write down a 5-dimensional (5D) metric, $\widetilde{g}_{ab}$, and apply the machinery of general relativity to it. We adopt roman indices to indicate a range over 5 dimensions, and keep greek indices to range over the 4 dimensions of spacetime. A tilde indicates a 5D quantity. Therefore, let us write the 5D metric in terms of its components that distinguish the fifth coordinate:

$$\widetilde{g}_{\mu\nu} = g_{\mu\nu} + \phi^2 A_\mu A_\nu \quad , \quad \widetilde{g}_{5\nu} = \phi^2 A_\nu \quad , \quad \widetilde{g}_{55} = \phi^2 \tag{11}$$

Here, $g_{\mu\nu}$ is the 4D spacetime metric, A^μ is the electromagnetic vector potential, and ϕ is a scalar field new to physics (in 1921 and now). Normalizing constants multiplying A^μ and ϕ are suppressed. Essentially, the 4D spacetime metric $g_{\mu\nu}$ is framed by the electromagnetic vector potential A^μ. The unknown scalar field ϕ takes the spot at the corner.

To obtain the field equations, the 5D vacuum Einstein equations are evaluated (compare equation 4):

$$\widetilde{R}_{ab} - \frac{1}{2} \widetilde{g}_{ab} \widetilde{R} = 0 \tag{12}$$

Examining just the spacetime part of (12), we recover the modified 4D Einstein equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{1}{2}\phi^2 T^{EM}_{\mu\nu} + \frac{1}{\phi}(\nabla_\mu \nabla_\nu \phi - g_{\mu\nu} \Box \phi) \tag{13}$$

where $T^{EM}_{\mu\nu}$ is the standard electromagnetic stress energy tensor, ∇_μ is the covariant derivative, and $\Box$ is the 4D d'Alembertian, $\nabla^\mu \nabla_\mu$. It is quite remarkable that the 5D Einstein equations should produce exactly this form, which is otherwise a hypothesis in standard general relativity. The electromagnetic stress-energy tensor is actually the 4D projection of a 5D vacuum. We also see in (13) a stress-energy tensor for the scalar field.

The equation (13) is obtained only under a major assumption, an additional cornerstone of the theory, in fact. It is the assumption that no field depends on the fifth coordinate: $\partial \widetilde{g}_{ab} / \partial x^5 = 0$. This famous restriction is called the *cylinder condition*. If derivatives with respect to the fifth coordinate are retained, (13) becomes quite complex. Even with the cylinder condition, a rich set of physics that maps well to our known world is recovered. Without the cylinder condition, there would be so many degrees of freedom in the theory that it could not be legitimately constrained.

The corresponding Friedmann equation of (13) is:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\left[\phi^2(t)\mathscr{E}_{EM}(t) + \mathscr{E}_\phi(t)\right] \tag{14}$$

where $\mathscr{E}_{EM}$ is the universal electromagnetic energy density, the time-time component of the electromagnetic stress energy tensor. The energy density of the scalar field, determined from the time-time component of its stress-energy tensor, is written as $\mathscr{E}_\phi$.

In fact, the correct field equations for the Kaluza hypothesis (11) were not written for another 25 years – Kaluza had not correctly handled the scalar field. The result given above in (13) was not obtained by Kaluza, but is attributed to Thiry in 1948 [5], and to Jordan at about the same time. (English translations of papers by Kaluza and Thiry became available only in 1987 [6].) But it was immaterial because already in the early 1920s, the theory was abandoned. To invoke an extra dimension only to throw it away with the cylinder condition seemed awkward. And the bothersome scalar field had no obvious candidate in nature. At around this time, a sea-change in physics was under way that would undermine the prospect of any classical unified field theory.

8. INTERLUDE: THE QUANTUM REVOLUTION – 1925

After percolating in physics for 25 years with the work of Planck on the blackbody spectrum, of Einstein on the photo-electric effect, and of Bohr on the atom, the quantum revolution exploded in physics in 1925. Around this time, Heisenberg and Schroedinger published nearly-simultaneous alternative descriptions of the wave mechanics of matter. These theories corrected the limitations of the classical electrodynamics of Maxwell, and set the direction of physics for the next century.

Around the same time, Klein published a quantum interpretation of the Kaluza theory [7]. The fifth dimension was explained as microscopic and wound in a circle, with a characteristic radius of 10^{-30} cm. At this point, the Kaluza theory forked, and was taken in a decidely quantum direction. In such a context, the early work is called "Kaluza-Klein". The idea of microscopic dimensions being used to attain a unified field theory has been an area of fertile exploration of physics since 1925 and early approaches to the strong and weak interactions involved general relativity in higher dimensions.

By the 1940s, with a successful quantum theory of electrodynamics in hand, and one for the strong and weak forces in promise, it was expected a quantum theory of gravity would obtain as well. It is perhaps for this reason that the discovery of the correct field equations [5] for Kaluza's hypothesis (11) was passed with little notice. Its unification of gravity and electromagnetism must have looked naive compared to quantum electrodynamics.

9. BRANS-DICKE THEORY – 1961

All of the foregoing history has been theoretical: general relativity, the cosmological constant, the Kaluza hypothesis. The last theoretical work was done by Brans & Dicke [8], who examined the general form of a scalar tensor theory of gravity, and a modification of the Einstein equations that looks like:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{1}{\psi} T^M_{\mu\nu} + T^\psi_{\mu\nu} \tag{15}$$

They hypothesize a scalar field ψ, whose inverse acts as the gravitational constant. (15) also contains a stress-energy from the scalar field. Brans & Dicke assumed ψ was driven by the mass-density of the universe, motivated by the observed correspondence between the value of the gravitational constant, the mass of the universe, and its size.

Brans & Dicke formulated their somewhat contrived theory as part of a broader investigation of Mach's Principle and alternative theories of gravity. It reinforced in the literature the idea of a variable gravitational constant, even though the work of Thiry contained it implicitly. And it established scalar-tensor theories as valid extensions of general relativity. The manifestation of (15) in the Friedmann equation will be qualitatively similar to (14).

And that is where the theory of cosmological, and therefore classical, scalar fields rested before the great discoveries of cosmology.

10. DISCOVERY: DARK MATTER – 1933, 1970

Dark matter may be considered the first great mystery of cosmology. It goes back to 1933 with observations of galaxy clusters [9]. The orbital speed v_G of galaxies in the cluster seemed too large for the visible mass M_V of the cluster to gravitationally bind it to a radius R_G. To compensate, an additional dark mass M_D is added to the energy balance equation:

$$\frac{1}{2}v_G^2 = \frac{GM_V}{R_G} + \frac{GM_D}{R_G} \tag{16}$$

As telescope resolution improved, the same effect was detected in 1970 in the individual stars in galaxies [10]. The rotational speed v_s of stars in the galaxy seems too large to be balanced by the visible mass M_V of the galaxy, suggesting a compensation from dark matter:

$$\frac{v_s^2}{R_G} = \frac{GM_V}{R_G^2} + \frac{GM_D}{R_G^2} \tag{17}$$

The effect is also seen in the hot, x-ray emitting gas of galaxy clusters. The pressure gradient of the hot gas seems to be balanced by an additional force attributed to dark matter

$$\frac{dP}{dr} = -\frac{G\rho_g M_V}{r^2} - \frac{G\rho_g M_D}{r^2} \tag{18}$$

Although it is spoken of as unseen, gravitating matter, we choose to consider dark matter as possibly a scalar field. For now, it is merely the parameterization of an unknown agent in the force balance equation for stars in galaxies, and galaxies in clusters.

11. DISCOVERY: INFLATION – 1980

Inflation is a deduced effect. We cannot see or measure the inflation force field, but a strong consensus for it has developed since its introduction 35 years ago. It was postulated to resolve at once 3 different puzzles that had already emerged in cosmology [11]. These puzzles were called "problems", and have been given famous names: the horizon problem, the flatness problem, and the monopole problem.

Without going in to the details of these 3 problems "solved" by the inflation hypothesis, suffice to say that all 3 problems seemed to imply that the observable universe inflated rapidly a split second after the Big Bang. Essentially, a brief episode of exponential expansion ensued, ala equation (10), from 10^{-36} s to 10^{-32} s after the Big Bang.

The expansion is parameterized as a scalar field that emerges from the vacuum to dominate the universe as an effective cosmological constant in an instant after the Big Bang. Then, its work done, the scalar field recedes into the vacuum, never to again participate in the dynamics of the universe. A field equation is hypothesized for the inflation scalar field that is tuned just so, to produce the desired behavior. While we are relatively sure that inflation happened, the nature of the scalar field, or of whatever field drove the inflation, remains mysterious.

12. DISCOVERY: DARK ENERGY – 1998

Observing platforms like the Hubble Space Telescope have realized what would have been an oxymoron 30 years ago: precision cosmology. That means that quantities like the Hubble constant, age of the universe, and cosmic microwave background temperature, are all known to several significant figures. It was that zeal for precision that led two teams in the 1990s to attempt to measure the deceleration of the expansion of the universe; that is, the degree to which the Big Bang is slowing under gravity.

They found the opposite: that the universal expansion is accelerating.[12], [13] The data behind this conclusion were exquisite: supernovae at $z \sim 0.5$ are about $1/4$ magnitude fainter than they would be if the universal expansion were not accelerating. Since the initial discovery, this result has been confirmed, and established the last missing piece of the universal energy budget. This expansive force driving cosmic expansion has been called *dark energy.*

Again, as we saw with inflation, the mathematical description of dark energy resorts to equation (10). Accordingly, dark energy is understood to behave, at least mathematically, like a cosmological constant. In fact, dark energy accounts for fully 70% of all the gravitating mass-energy in the universe. It's remarkable that we only discovered in 1998 what the universe is mainly made of.

A key characteristic of a cosmological constant is that it has negative pressure of magnitude equal to its energy density. Our precision cosmology allows us to measure the relation between dark energy pressure and energy density. A review in 2008 showed that $P_D = -(0.94\pm0.1)\mathscr{E}_D$.[14] Even allowing that dark energy does not have to be a cosmological constant, it sure looks like one. Therefore, this identity is now assumed in the standard model of cosmology, called Λ-cold-dark-matter (ΛCDM).

13. ΛCDM

In the age of precision cosmology, we are in a position to tabulate the items in the energy budget of the universe. Furthermore, it all adds up to what we observed in the Hubble "constant", so we know that nothing is missing.

First is the visible matter we've been aware of all along: all the stars and galaxies. It contributes an energy density $\mathscr{E}_{VM}$ that currently amounts to just 5% of the mass-energy of the universe.

Next is the dark matter, that mysterious stuff that seems to bind the galaxies. It contributes an energy density $\mathscr{E}_{DM}$ that currently amounts to about 27% of the mass-energy of the universe. Although the visible and dark matter provide just a third of the universe mass-energy today, they dominated the energy budget of the universe in the past, from about 50 thousand years until 10 billion years after the Big Bang.

Then we come to radiation, $\mathscr{E}_{rad}$. This categorizes all the massless particles in the universe: photons and neutrinos. The photons include the energy of the cosmic microwave background plus starlight. Taken together, the photons and neutrinos make up less than 10^{-4} of the total mass-energy of the universe. But there was a time when radiation dominated the universal energy budget, during the first 50 thousand years after the Big Bang.

Inflation is never tabulated, because it is not visible today, and existed only for an instant at the Big Bang. But it, too, once dominated the universe at a very early epoch. As it is modeled with a scalar field, we keep $\mathscr{E}_I$ in our budget.

When we come to dark energy, we finally get to the bulk of the universal mass-energy. Fully two thirds of the universal mass-energy is dark energy. And this dark energy appears to be a cosmological constant, Λ: it has negative pressure of the right magnitude. For the last few billion years, the Hubble expansion (6) has been driven primarily by dark energy, and it will apparently be so until the end of time.

Let us bring it all together in the total Friedmann equation:

$$H^2 = \frac{8\pi G}{3c^2}\left[\mathscr{E}_{VM} + \mathscr{E}_{DM} + \mathscr{E}_{rad} + \mathscr{E}_I\right] + \frac{\Lambda_D}{3} \qquad (19)$$

And, since quantum theory predicts a vacuum energy that would appear like a cosmological constant (constant everywhere in space), a cosmological constant is not unexpected. But there is a problem: the vacuum energy predicted by quantum theory is 120 orders of magnitude larger than the observed dark energy. It's a spectacular failure of theory. Obviously, something is wrong with our understanding of the vacuum and with quantum theory on cosmological scales. We expect something like a cosmological constant, but we don't understand dark energy.

So, of the energies operating in our universe on the largest scales, we understand only two of the five contributors to (19): visible matter and radiation. Together, about 5% of everything. When we ask what we know about the other 3 forces, we can only speak about how we parameterize them.

- Inflation: a scalar field of unknown origin.
- Dark energy: a constant scalar field of unknown origin – or maybe the vacuum of unknown physics.
- Dark matter: for all we know, a scalar field of unknown origin.

14. BACK TO THE FUTURE: KALUZA UNIFICATION

From its modest beginnings as an unwanted side-effect of the original Kaluza theory, to its dominance of observational cosmology, the scalar field has seen a remarkable rise in physics over the past century. Conversely, the hopes for a quantum theory of gravity have sunk, the best minds of the 20th century unable to scale that rampart. The failure of quantum gravity is all the more frustrating, given that the quantum electromagnetic force has been formally unified with the weak quantum force.

In light of these facts, a return to the unified theory of Kaluza seems in order. It provided the only successful unification of general relativity with electromagnetism, covering the only two long-range forces known in the universe. Furthermore, the theory predicts a third, scalar long-range force. Other scalar field theories seem ad hoc by comparison.

Going back to the reasons for the abandonment of the classical Kaluza unification in the 1920s, they do not now seem so compelling: the rush to quantum gravity may have been premature, and the scalar field is now welcome, thanks to the discoveries of cosmology. Given that we have as many as 3 new scalar fields to account for, the one that provides the fulcrum in the unification of gravity and electromagnetism seems worth investigating. Taking the 3 unknown scalar fields as possible evidence for the Kaluza scalar field, we consider the implications for interstellar travel from such a unification of gravity and electromagnetism.

15. 5-DIMENSIONAL MOTION

General relativity assembles several independent hypotheses. One of these is the Einstein equations (4) for the metric. Another, closely related hypothesis is to provide the form of the stress-energy tensor in (4) – general relativity contains no prescription. Fortunately, many standard forms have been developed for many situations. A third, completely independent, hypothesis is the geodesic hypothesis that yields the equations of motion in general relativity. The geodesic equation complements the Einstein equations to provide a complete mathematical description of gravity.

Kaluza's original paper examined all 3 independent aspects of general relativity in 5 dimensions: the field equations, the geodesic equation, and the form of the stress-energy tensor. It was considered almost miraculous that the single ansatz (11) produced the Einstein equations with electromagnetic sources (13), the Maxwell equations from the 5D Einstein equations (the $\mu5$ component of equation 12, (25)), and the Lorentz force equation from the 5D geodesic equation (22). From the combined picture he was able to deduce the remarkable result of his hypothesis: that electric charge arises from motion in the 5th dimension. Let us see why this is so.

The hypothesized metric (11) implies an invariant 5D length element ds:

$$ds^2 \equiv \widetilde{g}_{ab}dx^a dx^b = g_{\mu\nu}dx^\mu dx^\nu - \phi^2(kA_\nu dx^\nu + dx^5)^2 \tag{20}$$

There is a corresponding "5-velocity" vector $\widetilde{U}^a \equiv dx^a/ds$.

The 4D equations of motion in this theory are obtained from a 5D geodesic equation in terms of $\widetilde{U}^a$:

$$\widetilde{U}^b \widetilde{\nabla}_b \widetilde{U}^\mu = \frac{d\widetilde{U}^\mu}{ds} + \widetilde{\Gamma}^\mu_{ab}\widetilde{U}^a \widetilde{U}^b = 0 \tag{21}$$

where the 5D connections $\widetilde{\Gamma}^a_{bc}$ are constructed as in 4D from derivatives of $\widetilde{g}_{ab}$. Now use (11) to recast (21) as

$$\frac{d\widetilde{U}^\mu}{ds} + \Gamma^\mu_{\alpha\beta}\widetilde{U}^\alpha \widetilde{U}^\beta = k\phi^2 \widetilde{Q} g^{\mu\beta} F_{\beta\alpha}\widetilde{U}^\alpha - \frac{\widetilde{Q}^2}{2}\partial^\mu \phi^2 \tag{22}$$

where $\widetilde{Q} \equiv -(\widetilde{U}^5 + A_\alpha \widetilde{U}^\alpha)$. Equation (22) has been studied in various forms by many authors [15–18].

Equation (22) is the generalized equation of motion in 4D, the extensions to the known laws of physics, as it were. It has the generalized term quadratic in the proper velocity, as in the 4D geodesic equation, and the term linear in the proper velocity, as for the Lorentz force equation. But it is written in terms of 5-velocities. It is a compact form that obviously has the Lorentz force law and the geodesic equation built in, but it takes some work to unpack it in standard terms.

To make contact with standard theory for 4-velocities, cast (21) in terms of $U^a \equiv dx^a/d\tau$, where $d\tau^2 \equiv g_{\mu\nu}dx^\mu dx^\nu$ and $U^\mu \equiv dx^\mu/d\tau$:

$$\frac{dU^\mu}{d\tau} + \widetilde{\Gamma}^\mu_{\alpha\beta}U^\alpha U^\beta + 2\widetilde{\Gamma}^\mu_{5\alpha}U^\alpha U^5 + \widetilde{\Gamma}^\mu_{55}(U^5)^2 + U^\mu \frac{d}{d\tau}\ln\left(\frac{cd\tau}{ds}\right) = 0 \tag{23}$$

The form (23) is actually independent of any choice of the 5D metric. But it demonstrates which connection is necessary to recover the Lorentz force law: the one linear in the 4-velocity U^ν. In standard theory, the effects of gravity in the equations of motion are independent from the Lorentz force law for electromagnetic effects on charged particles. The 5D hypothesis elegantly unites them.

It turns out that with the metric (11), $\widetilde{\Gamma}^\mu_{5\alpha}$ is indeed proportional to the Maxwell tensor, so that the term linear in U^α on the left hand side of (23) can account for the Lorentz force law *if* the component of velocity in the fifth dimension U^5 is identified with electric charge q:

$$U^5 \equiv \frac{dx^5}{d\tau} \Longrightarrow \frac{q}{mc} \tag{24}$$

In other words, electric charge arises from "motion" in the 5th dimension.

16. IMPLICATIONS FOR INTERSTELLAR TRAVEL

The engineering problem of interstellar travel has two aspects: there is a fuel problem, and there is a time-distance problem. The fuel problem presumes conventional propulsion technology and conventional, sub-relativistic speeds. Once a workable engine is designed, no matter how feeble, then the problem of interstellar travel reduces to the problem of providing fuel for the engine.

It is the time-distance problem that is of interest to us here. The time-distance problem is that we can never go faster than light, and that the stars are so far that even at the speed of light, it would take eons to reach them as measured by clocks in the civilization back home. Therefore no civilization can become space-faring to the stars. They could only send out emissaries like messages in bottles, never to return. Solving the time-distance problem is necessary to realizing *feasible* interstellar travel – and probably feasible interplanetary travel as well. This latter speculation is based on the expectation that a solution to the time-distance problem solves the fuel problem as a matter of course. Therefore, for the purposes of pursuing the dream of interstellar travel for our species, no problem is of higher interest than the time-distance problem. And that is why the potential discovery of the Kaluza scalar field is so encouraging.

The time-distance problem is not an engineering problem, it is a physics problem. Practical interstellar travel will require new physics. The mystery fields of modern cosmology may herald that new physics. And this is not idle speculation. Rather, it is based on the recognition that an as-yet undiscovered scalar field may be the missing field to unify gravity and electromagnetism. Once we couple gravity to electromagnetism, a force we control, then we control gravity.

Recall that the equations from Kaluza's unified theory are based on the 5D Einstein equations in vacuum (12), and that we could get the usual equations of general relativity (13) with an electromagnetic stress tensor by considering the spacetime components ($\mu\nu$) of (12). Looking closely at (13), we see that the coupling of the electromagnetic stresses $T^{EM}_{\mu\nu}$ to gravity is modulated by the scalar field ϕ. Conventional general relativity would tell us that all energy sources couple equally to gravity, but the unified field theory seems to predict that the coupling of electromagnetic energy to gravity can be modified, if the scalar field can be modified. If the scalar field could be modified, this would portend anomalous gravitational effects from electromagnetic configurations. Let us see about control of the scalar field.

We already mentioned that the same package (12) also contains the (modified) Maxwell equations when we consider the $\mu 5$ component:

$$g^{\alpha\beta}\nabla_\beta(\phi^{-3}F_{\nu\alpha}) = 0 \tag{25}$$

where $F_{\alpha\beta} \equiv \partial_\alpha A_\beta - \partial_b A_\alpha$ is the traditional Maxwell field strength tensor.

When we consider the vacuum equation $\widetilde{R}_{55} = 0$, we obtain the equation for the as-yet-unknown scalar field ϕ:

$$\Box\phi = \frac{1}{4}\phi^3 F^{\alpha\beta}F_{\alpha\beta} \tag{26}$$

Both equations were first obtained by [5]. We see the vacuum Maxwell equations in (25) – but modified by the scalar field. And we see the field equation for ϕ in (26) – with an electromagnetic source! So the scalar field is coupled to electromagnetism, and it is a matter of engineering to determine from these equations what sorts of scalar field variations can be achieved through manipulation of the electromagnetic field. Then, such variations could in turn be used to adjust the coupling of gravity to electromagnetism.

In other words, if one of the cosmological mystery fields is the Kaluza scalar field, it would portend *electromagnetic gravity control*. But there is more.

At its essence, the Kaluza hypothesis involves a fifth dimension. As Kaluza and Einstein conceived it, the fifth dimension is macroscopic. The cylinder condition can be understood as a boundary condition on the equations, one based on the world we inhabit. We don't perceive the fifth dimension, so it must not vary. It's closely related to the fact that an observer in inertial motion cannot detect that motion. It could be said that everything is moving inertially through the fifth dimension.

This is quite different than the quantum interpretations that arose after 1925, including Klein's famous introduction of a microscopic fifth dimension. It makes all the difference for our purposes, because we seek a hyperspace dimension as new physics that could possibly solve the time-distance problem. Our current understanding of the limiting speed of light is based on a fundamental feature of spacetime: that the spacetime interval $d\tau^2 \equiv g_{\mu\nu}dx^\mu dx^\nu$ is coordinate invariant. This is the root of why nothing can be accelerated faster than light.

With the Kaluza theory, we have a hyperspace dimension, as described by (20). The question is to what degree fixed 5D distances can be traversed with different spacetime intervals. It is only such profound modifications of current physics that could surmount the light barrier and realize the dream of practical interstellar travel.

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