Hydrodynamic assessment of avalanche impact load

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A method is described for assessing the impact load from an avalanche. Instead of considering the problem from a force-impulse perspective, the avalanche is treated as a fluid and the impact is ascribed to hydrodynamic pressure. The calculation will be done in 3 iterations in order to illustrate and isolate the various factors, with each iteration increasing in complexity.

1. Vertical fall onto horizontal roof

Consider the simple case of snow dropped from a height H. It will accelerate under the influence of gravity and attain a velocity V at the time it strikes the horizontal roof. The snow has a mass density ρ. The acceleration under gravity is g.

Hydrodynamic flows are characterized by a dynamic pressure $P_d = \rho V^2$. Conversely, static fluids of a depth D provide a static pressure of $P_s = \rho g D$. This and subsequent calculations will compare the static load on a roof bearing snow of depth D with the "impact load" imparted by a hydrodynamic flow with velocity V.

Snow accelerating under gravity from a height H will attain a velocity $V = gt$, where t is the time to fall H. The fall distance $H = gt^2/2$. These are standard results for velocity and distance under constant acceleration. Combining these two we find:

$$V=(2Hg)^{1/2}$$

and the hydrodynamic pressure on the roof is:

$$P_d = 2\rho g H.$$

Interestingly, the dynamic and static pressures both are proportional to ρg. Therefore define a critical depth D_c of snow on the roof which is the depth at which the hydrostatic load equals the impact load:

$$\boxed{D_c = 2H.}$$

This means the depth of snow on the roof necessary for a static load to equal the impact load is twice the height the avalanche falls, testament to the enormity of the dynamic pressure relative to the static pressure. Even a modest drop of 100 feet would have an equivalent static load corresponding to a depth of snow on the roof of 200 feet.

2. Fall down a frictionless slope onto a horizontal roof

Now consider the more realistic case of snow falling down a slope making an angle α with the horizontal. A flat slope corresponds to $\alpha = 0$ and a vertical one to $\alpha = 90$ degrees. In this case, the snow is accelerated downhill by the component of *g* parallel to the slope, $g\sin\alpha$.

It is assumed here that the slope is frictionless. My intuition tells me that this is a pretty good approximation for a number of regimes: snow sliding over a hard slab, snow at the slide interface fluidized by the slide, and snow riding over a layer of air churned into the slide much like the puck sliding on an air hockey table. In other words, the frictional deceleration *should be* much less than the gravitational acceleration. I expect this assumption to break down for long runouts. At the end of a big slide it will be plowing snow on shallower slopes. This resistive force from stable snow will limit the velocity achieved by snow in a slide.

Now, $V=tg\sin\alpha$ and the distance along the slope $H = g\sin\alpha\, t^2/2$ so that

$$V=(2Hg\sin\alpha)^{1/2}$$

However, the snow is not impacting normally to the roof in this case. No hydrodynamic stress is imparted to the roof from the component of snow velocity parallel to the roof. The expression ρV^2 can be understood as a momentum flux density: a momentum density of ρV is delivered by the flow at a rate V. Due to the slope angle, both the momentum density and the flow rate are attenuated by a factor of $\sin\alpha$. This implies that the hydrodynamic pressure delivered normal to the roof is:

$$P_d = 2\rho g H\sin^3\alpha$$

and the critical depth at which hydrostatic pressure equals dynamic pressure is

$$\boxed{D_c = 2H\sin^3\alpha.}$$

The slope strongly attenuates the dynamic pressure delivered normal to the roof compared to the case of a vertical drop. One factor of $\sin\alpha$ results from the reduced gravitational acceleration along the slope, one factor from the diminished momentum delivered normal to the roof, and one factor from the diminished rate of delivery of momentum. Some values of this function are tabulated below.

α	$\sin^3\alpha$
30	0.13
45	0.35
60	0.65
75	0.90

This means snow falling 100 feet down and along a 30 degree slope would impart a pressure equivalent to a static snow load on the roof 26 feet deep. If the slope were 45 degrees, the critical hydrostatic depth would be 70 feet. The impact load is greatly diminished for low grade avalanches but is still high for steep slope slides.

3. Fall down a frictionless slope onto a pitched roof

Consider now a roof pitched at an angle β with the horizontal, so that the roof normal is turned away from the slide velocity vector to further minimize the hydrodynamic pressure imparted normal to the roof. We will consider here the case of an A-frame roof with the plane of the "A" shape parallel to the slope surface. This is the case that minimizes impact load on the roof since turning the roof surface into the slide velocity vector will increase the impact load normal to the roof. More complex, multi-faceted roof planes are possible but their effect is captured by this simple case.

The avalanche velocity vector is still as given in section 2 above, $V=(2Hg\sin\alpha)^{1/2}$. In order to compute the dynamic pressure imparted to the sloped roof, find the projection of the slide velocity vector onto the roof normal vector. In a Cartesian **x-y-z** coordinate system, the slide velocity vector **v** can be assumed in the **x-z** plane, and written

$$\mathbf{v} = \cos\alpha\ \mathbf{x} - \sin\alpha\ \mathbf{z}$$

Likewise, the roof normal $\mathbf{R}_N$ can be assumed to be in the y-z plane, and written

$$\mathbf{R}_N = \sin\beta\ \mathbf{y} + \cos\beta\ \mathbf{z}$$

Therefore the projection of these two vectors is simply $(-\sin\alpha\ \cos\beta)$. The negative sign is because the two vectors point in opposite directions. In section 2 above, the momentum delivered to the roof and the rate at which it was delivered were both attenuated by a factor of $\sin\alpha$. Here the slope of the roof modifies the factor to be $\sin\alpha\cos\beta$. If the roof plane is vertical (β = 90 deg), then no dynamic load is exerted on the roof. If the roof plane is horizontal (β = 0 deg), then the result of section 2 is recovered.

The dynamic pressure delivered to an A-frame roof with the plane of the "A" parallel to the slope surface is then

$$P_d = 2\rho g H\sin^3\alpha\ \cos^2\beta$$

The critical depth at which hydrostatic pressure equals dynamic pressure is

$$D_c = 2H\sin^3\alpha\cos^2\beta\ .$$

Obviously, a pitched roof further reduces the dynamic load of a slide. Some values are shown below:

β	$\cos^2\beta$
30	0.75
45	0.5
60	0.25
75	0.07

Some values of $\sin^3\alpha\cos^2\beta$ for various slope and roof angles are tabulated below

α \ β	30	45	60	75
30	0.098	0.065	0.033	0.0091
45	0.27	0.18	0.088	0.025
60	0.49	0.33	0.16	0.046
75	0.68	0.45	0.23	0.063

The values quantify how the dynamic load of increasingly steep slopes can be mitigated by increasing roof pitch. Judicious choices of roof pitch can keep the dynamic load equivalent static depth below 20% of the slope height (values shown in green), but impractically steep pitches are required to get much below this. For example, a roof pitched at 75 degrees would require 37 feet of vertical height for every 10 feet of horizontal coverage.

In summary, dynamic loads are enormous compared to static loads and perhaps only strong materials can withstand impact for long, steep avalanche slope exposure. It is worth noting that this calculation is independent of the thickness of the avalanche slab.