Five-Dimensional Relativity *and the* Nature of the 5th Dimension

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Engineering Motivation for Seeking a Breakthrough

According to special relativity, interstellar exploration is impossible for a planet-bound civilization

(but not impossible for no-return explorers)

1. The speed of light is too slow

No object can be accelerated beyond the speed of light. Traveling at the speed of light would require 3 years to the nearest star, and 100,000 years to cross the galaxy (as measured in the rest frame of the galaxy).

$$\frac{dx}{dt} \leqslant c$$

2. Travelers are disconnected in time

Time dilation effects accrue which isolate the traveler temporally from the home planet. While a traveler accelerated at 1 g for 5 years, 74 years would pass on the home planet.

$$t_{home} \propto e^{a t_{trav}/c}$$

3. The gravitational constant is too small

Warp-drive or wormhole distortions of spacetime that might mitigate the two problems above cannot be realized with terrestrial amounts of mass energy. Jupiter masses, at least, are required to bend spacetime on engineering scales.

$$a = G\frac{M_J}{r^2}$$

Observational Motivation for Seeking a Breakthrough

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\left[\epsilon_{rad} + \epsilon_{Mdark} + \epsilon_{Mvis} + \epsilon_{infl}\right] + \frac{\Lambda_D}{3}$$

?

$\epsilon_{infl} \propto \delta(t - t_I)$	Inflation scalar field dominant 10^{-35} seconds after the Big Bang	
$\epsilon_{rad} \propto \frac{1}{a^4}$	Radiation dominant first 50,000 years	$\Omega_{rad} = 8\times 10^{-5}$
$\epsilon_{dark} \propto \epsilon_{vis} \propto \frac{1}{a^3}$	Matter dominant from 50K to 10B years	$\Omega_{Mdark} = 0.27$ $\Omega_{Mvis} = 0.05$
$\Lambda_D \propto \text{const}$	Dark energy dominant after 10B years (now)	$\Omega_D = 0.68$

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5D General Relativity: *Field Equations*

$$\tilde{R}_{ab} - \frac{1}{2}\tilde{g}_{ab}\tilde{R} = 0$$

$$\tilde{g}_{ab} = \begin{pmatrix} g_{\mu\nu} + k^2\phi^2 A_\mu A_\nu & k\phi^2 A_\mu \\ k\phi^2 A_\nu & \phi^2 \end{pmatrix}$$

$$k^2 = 16\pi G/c^4$$

$$\frac{\partial \tilde{g}_{ab}}{\partial x^5} = 0$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}\phi^2 T^{EM}_{\mu\nu} + T^{\phi}_{\mu\nu}$$

$$\nabla^\mu F_{\mu\nu} = -3F_{\mu\nu}\partial^\mu \ln\phi$$

$$\nabla^2\phi = \frac{4\pi G}{c^4}\phi^3 F_{\mu\nu}F^{\mu\nu}$$

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5D General Relativity: *Equations of Motion*

$$\tilde{U}^b \tilde{\nabla}_b \tilde{U}^a = 0$$

$$ds^2 \equiv \tilde{g}_{ab} dx^a dx^b \qquad \tilde{U}^a \equiv \frac{dx^a}{ds}$$

$$\tilde{g}_{ab} = \left(\begin{array}{c|c} g_{\mu\nu} + k^2 \phi^2 A_\mu A_\nu & k\phi^2 A_\mu \\ \hline k\phi^2 A_\nu & \phi^2 \end{array} \right)$$

$$k^2 = 16\pi G/c^4$$

$$\frac{\partial \tilde{g}_{ab}}{\partial x^5} = 0$$

$$U^\alpha \nabla_\alpha U^\nu = k\phi^2 Q g^{\nu\beta} F_{\beta\alpha} U^\alpha + \frac{1}{2} Q^2 \partial^\nu \phi^2 - U^\nu \frac{d}{d\tau} \ln\left(\frac{c\, d\tau}{ds}\right)$$

$$Q \equiv U^5 + kA_\nu U^\nu$$

$$c^2 d\tau^2 \equiv g_{\mu\nu} dx^\mu dx^\nu \qquad U^\mu \equiv \frac{dx^\mu}{d\tau}$$

$$\phi^2 Q \frac{c\, d\tau}{ds} = const$$

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History of Development of 5D Relativity

- Original idea from Kaluza (1921), but assumed constant scalar field, so field equations were incomplete. Yet all key ideas were present.
- Klein (1925) mated Kaluza's hypothesis to quantum physics, and introduced the idea of a compact 5th dimension. This was a major fork of Kaluza theory, and became known as “Kaluza-Klein”
- Einstein & colleagues did a lot of work extending Kaluza's idea in various directions in the 1930s. Bergmann (1942) and Bargmann (1957) also wrote major reviews of the basic theory.
- Thiry (1948) obtained the full, self-consistent field equations for Kaluza's 5D metric
- Jordan & colleagues are said to have done much work with the theory circa 1950, also obtaining the full field equations, and some authors speak of “Brans-Dicke-Jordan” scalar field theory
- Pauli has a treatment in his 1958 relativity text
- Gross & Perry (1983), Gegenberg & Kunstatter (1984)
- Applequist, Chodos, & Freund (1987) provided English translations of Kaluza (1921) and Thiry (1948) – there are still no translations of Jordan, and some of his early work appears to have been lost
- Wesson & colleagues did a lot of work with the theory in the 1990s, relaxing the cylinder condition, and emphasizing the 5th dimension need not be compact

The Nature of the 5th Dimension

$$k\phi^2 U^5 \rightarrow \frac{q}{mc}$$ to fix identification of the Lorentz force law

Electric charge is the manifestation of motion in the 5th dimension

$$\frac{\partial \tilde{g}_{ab}}{\partial x^5} = 0$$ since we don't observe any obvious fields that result from the relaxation of the cylinder condition

The fifth dimension is macroscopic but invisible because there is no local variation of fields in that dimension

Gravity Control

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} \phi^2 T^{EM}_{\mu\nu}$$

A standard feature of scalar-tensor theories

$$\nabla^2 \psi = 4\pi G \phi^2 \rho_{EM}$$

In this case, the control is electromagnetic

$$\nabla \psi \equiv f_G = -\phi^2 G \frac{E_{EM}/c^2}{r^2}$$

Gravity Control in Brans-Dicke Theory

A scalar field assumed to depend on the mass density of the universe

$$\nabla^2 \phi = \beta T^M$$

Identified with the gravitational constant

Enters the field equations of gravity, contributing to spacetime curvature

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{1}{\phi} \alpha T^M_{\mu\nu} + T^\phi_{\mu\nu}$$

Assumed *not* to enter the equations of motion

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Cliff Will PPN Evaluation

$$S = \frac{1}{16\pi} \int \left[\varphi R - \frac{\omega(\varphi)}{\varphi} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + 2\varphi \lambda(\varphi) \right] \sqrt{g}\, d^4 x + S_{NG}$$

Time delay and light deflection: $\omega > 500$

not quite
the same

$$S_{5D} = \int \phi^2 \left[R - \frac{2}{\phi} g^{\mu\nu} \nabla_\mu \partial_\nu \phi - \frac{k^2 \phi^2}{4} F^{\mu\nu} F_{\mu\nu} \right] \sqrt{g}\, d^4 x$$

$$= \int \phi^2 \left[R - \frac{2}{\phi^2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - \frac{k^2 \phi^2}{4} F^{\mu\nu} F_{\mu\nu} \right] \sqrt{g}\, d^4 x$$

Does 5D Relativity Address the Engineering Motivation?

Yes!

- It extends the limiting spacetime of special relativity
- It offers a prospect for gravity control at terrestrial scales
- It describes a hyperspace dimension

Does 5D Relativity Address the Observational Motivation?

Yes!

- It predicts a scalar field
- It lies roughly in the class of scalar field theories that are considered plausible extensions of general relativity, but the unique couplings are such that this particular theory may not be constrained by standard PPN theory
- It provides the only successful unification of general relativity with another force field

Energy-Momentum-Charge 5-Vector

$$ds^2 \equiv \tilde{g}_{ab} dx^a dx^b \qquad \tilde{U}^a \equiv \frac{dx^a}{ds}$$

Time → Energy

Space → Momentum

5th dimension → Electric charge

$$\frac{d\tilde{U}_a}{ds} = \frac{1}{2} \tilde{U}^b \tilde{U}^c \frac{\partial \tilde{g}_{bc}}{\partial x^a}$$

Experimental Falsifiability

Electric charge is not a Lorentz scalar

$$\frac{dU^{\nu}}{d\tau} = \underbrace{\left(\frac{q}{mc} + \frac{16\pi G}{c^4} A_{\mu} U^{\mu}\right)}_{\text{constant of motion}} F^{\nu}_{\alpha} U^{\alpha}$$

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Conclusions

- 5D relativity is a promising extension of physical law to investigate
  - It describes gravity control
  - It accounts for a cosmic scalar field
  - It unifies general relativity and classical electromagnetism
- The 5th dimension is macroscopic
- Electric charge is the manifestation of motion in the 5th dimension
- Cliff Will's PPN evaluation of scalar-tensor theories does not appear to apply to the 5D theory, due to couplings between the scalar field and the EM field
- Experimental verification/falsification could be approached through the prediction that electric charge is not a true Lorentz scalar