Review of
Quantum Theory at the Crossroads: Reconsidering the 1927 Solvay Conference
by
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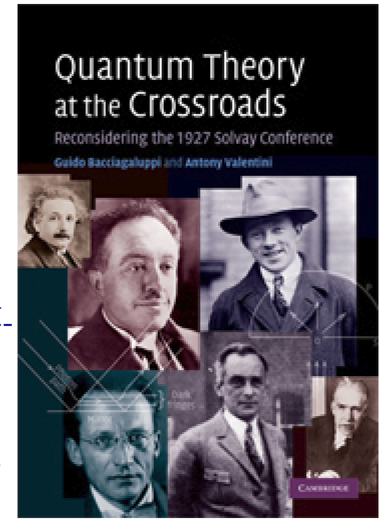

Quantum Theory at the Crossroads (QTC) is a fascinating follow-up for anyone who may have read about the Pilot Wave in my book, *The Spirit of Reason*. Bacciagaluppi and Valentini show how the real history of de Broglie's pilot wave proposal differs from the popular history. QTC provides an English translation of the presented papers and of the discussions at this pivotal meeting in the history of physics. The authors also provide a contextual overview of the state of physics leading up to the meeting, they provide a mathematical summary of de Broglie's pilot wave theory, they provide a description of the organization of the meeting and selection of the agenda, and they provide a modern discussion of the results presented at the meeting.

What emerges is that de Broglie presented an alternative quantum theory to the wave description of Schroedinger or the matrix description of Heisenberg and Born, and that his pilot wave theory was far more comprehensive and consistent in accounting for all phenomena than he is generally given credit for. At the meeting, all 3 views were presented by their authors (Born and Heisenberg shared their presentation), and the discussion centered around the 3 alternative theories.

In *The Spirit of Reason*, page 127, I wrote:

> At the 1927 Solvay Congress...de Broglie presented a pilot wave interpretation of the wave function. De Broglie suggested that the wave function was a real physical field that affected the motion of material objects. He proposed that it was a discovery of a kind with the electric and gravitational fields. Indeed, the Schroedinger equation can be recast to reflect this view. [true so far] De Broglie's interpretation withered under the direct objections raised by Pauli which he was unable to convincingly repudiate. [not true] It expired under the indifference of Schroedinger and Einstein...[not true].

In QTC it is discussed how the erroneous historical view repeated above is not born out by the records of the proceedings but rather, is based on the recollections of a small number of participants or even on the spotty recollections of de Broglie himself 40 years after the meeting. In *The Spirit of Reason*, I also repeat the recollection by Ehrenfest of “Bohr towering over everybody”, but in fact, Bohr did not formally present at the meeting. His participation in the recorded discussions is limited, and his submission to the published proceedings was not any presentation he gave or a summary of his discussion, but rather a paper he had previously published elsewhere.

The Spirit of Reason also repeats the view that David Bohm was the first to get the pilot wave theory right. In fact, de Broglie's 1927 work was entirely equivalent to Bohm's, just premised on velocity instead of acceleration (this will be discussed below). Bohm rediscovered the pilot wave for a later generation, re-popularizing the ideas de Broglie's generation had rejected. In the 1990s, the first pilot-wave based textbooks appeared. According to Bacciagaluppi and Valentini, the Copenhagen interpretation has long since lost its eminent position as “the” interpretation of the meaning of quantum theory.

De Broglie suggested a new dynamics based on the insight that the variational principle of Maupertis for corpuscular systems, and the variational principle of Fermat for wave phenomena, were different aspects of an underlying identity. In this new dynamics, the velocity of a particle is determined by the gradient of an underlying *phase wave*, later to be called the *pilot wave*. This is a radical departure from classical dynamics, wherein particle velocities are determined by forces.

Let's consider the de Broglie pilot wave theory in detail, following QTC.

The context for de Broglie's Nobel-Prize-winning discovery of the wave behavior of matter is to be found in Einstein's earlier photon hypothesis of 1905. Before Einstein's photon hypothesis, electromagnetic energy was believed to be best understood as a wave. Although the photo-electric effect was well-described with the photon hypothesis, the relation between the photon and the electromagnetic wave seemed mysterious.

In fact, few scientists took seriously the corpuscular nature of radiation implied by the photon hypothesis. For nearly two decades Einstein worked alone to reconcile the photon with the electromagnetic wave. Einstein proposed in 1909 that the light waves contain localized fragments of energy, the photons, presaging de Broglies proposal for waves of matter with localized fragments of mass. Another possibility was that the electromagnetic wave guided the photon.

In 1923 de Broglie published the basic ideas of his pilot wave theory, with which he predicted the wave nature of matter. De Broglie noted Maupertis' principle of least action for determining the trajectories of particles of mass m moving with velocity $\boldsymbol{V}$ between points *a* and *b*,

$$\delta \int_a^b m \boldsymbol{V} \cdot d\boldsymbol{x} = 0 \quad , \tag{1}$$

and found a correspondence with Fermat's principle for light rays,

$$\delta \int_a^b d\phi = 0 \quad , \tag{2}$$

where ϕ is the phase of the light ray between points *a* and *b*.

To make this connection, de Broglie started with a paradox implied by relativity. He first assumed that matter has an “internal periodic phenomenon”, a wave nature of some sort. Equating the rest energy of the particle, $m_0 c^2$, to the energy-frequency relation Einstein discovered for the photo-electric effect, $E = h\nu$, one calculates a “rest frequency” $\nu_0 = m_0 c^2 / h$. If the massive particle is moving with velocity *V*, then it will have relativistic energy $m c^2 = \gamma m_0 c^2$, where $\gamma \equiv (1 - V^2/c^2)^{-1/2}$, which implies a frequency $\nu = \gamma m_0 c^2 / h$. But since moving clocks run slower, the internal periodic phenomenon should have a time-dilated frequency of $\nu = \nu_0 / \gamma$. De Broglie demands that these two distinct oscillations should remain in phase, and finds this occurs for a fictitious wave of phase velocity $V_{ph} = c^2 / V$.

Now assume a generic form for the wave phase, $\phi \propto \boldsymbol{k} \cdot \boldsymbol{x} - \omega t$, where $\boldsymbol{k}$ and ω are the usual wavenumber and wave angular frequency so that $V_{ph} = \omega / k = \lambda \nu$. Then write for the spatial part of the phase:

$$\begin{aligned} d\phi = \boldsymbol{k} \cdot d\boldsymbol{x} &= \frac{\omega}{V_{ph}} dx \\ &= 2\pi\nu \frac{V}{c^2} dx \\ &= \frac{2\pi}{h} m V \, dx \end{aligned} \tag{3}$$

The last line is de Broglie's famous relation

$$\boldsymbol{p} = \hbar \boldsymbol{k} \quad . \tag{4}$$

Of course, de Broglie also extended Einstein's photo-electric equation to matter:

$$E = \hbar \omega \quad . \tag{5}$$

The curious thing about these phase waves is that they are plane waves, infinite in extent. Where is the localized particle? This was the same question Einstein faced when reconciling the photon with the electromagnetic wave.

For de Broglie, the particle is guided by the phase wave. The particles are singularities in the amplitude of the wave, singularities described by an as-yet-undiscovered deterministic theory. For de Broglie, the classical theory is a geometric-optics limit of the wave theory.

These results were written up as de Broglie's PhD thesis in 1924. His resolution of the apparent paradox of the time-dilation frequency and the kinetic energy frequency is called the `theorem of phase harmony'. He shows that the group velocity of the phase wave is the particle velocity. He notes that the phase velocities are greater than the speed of light, so the phase wave cannot be transporting energy.

In his thesis he considers the case of an electron of charge q and mass-energy m in an electrostatic potential ϕ , and establishes relations for the frequency and phase velocity of the phase wave:

$$\begin{aligned} \nu &= (m_0 \gamma c^2 + q\phi)/h \\ V_{ph} &= (\gamma m_0 c^2 + q\phi)/\gamma m_0 V \end{aligned} \tag{6}$$

These equations would be Schroedinger's starting point for his search for the equation of the phase wave.

In his thesis, de Broglie presents the relativistic guidance equations:

$$E = \hbar \partial \phi / \partial t$$
$$\boldsymbol{p} = -\hbar \nabla \phi \tag{7}$$

These equations constitute a new form of dynamics, in which momentum is imparted to a particle not from a force, but from a guiding wave. Dynamics in terms of forces is dubbed `second order dynamics'. De Broglie had introduced a `first order dynamics' which abandons Newton's first law. Relativity requires us to associate a phase wave with every moving body.

In a famous paper on the quantum theory of the ideal gas, Einstein calculated that fluctuations in the new Bose-Einstein statistics had two parts, which could be interpreted as particle contributions and wave contributions. Einstein argued that the wave contribution should be understood in terms of de Broglie's matter waves, citing de Broglie's thesis. Thus did de Broglie's work become known outside France.

One of those who read about de Broglie's work in Einstein's paper was Schroedinger. He set about to find the equation of de Broglie's phase wave. Schroedinger started with de Broglie's equations (6) and applied them to the case of the hydrogen atom with a Coulomb field of $\phi = -q/r$. Schroedinger eliminated *V* from the pair of equations (6) and recovered an equation for the phase wave speed $V_{ph}(\nu, r)$. Then Schroedinger proposed the standard wave equation for the phase wave Ψ :

$$\nabla^2 \Psi = \frac{1}{V_{ph}^2} \frac{\partial^2 \Psi}{\partial t^2} \tag{8}$$

Further assuming a time dependence of $\Psi \propto \exp(-2\pi i \nu t)$, Schroedinger wrote:

$$\nabla^2 \Psi = -\frac{4\pi^2 \nu^2}{V_{ph}^2} \Psi \tag{9}$$

Equation (9) was Schroedinger's first attempt at the equation for de Broglie's phase wave for the hydrogen atom. In fact, the predictions of (9) disagreed with experiment. By the time of the Solvay meeting, however, Schroedinger had discovered his famous equation for a non-relativistic particle of mass m_0 in a potential *U*:

$$\frac{-\hbar^2}{2\mathrm{m}_0} \nabla^2 \Psi + U \Psi = i\hbar \frac{\partial \Psi}{\partial t} \tag{10}$$

Equation (10) can be understood as the equation obeyed by a wave $\Psi \propto \exp(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)$, with a dispersion relation of $E=p^2/m_0$, and where *E* and *p* are given by de Broglie's equations (4) and (5).

Although Schroedinger's work departed from de Broglie's phase wave theory, Schroedinger abandoned the concept of particle trajectories guided by the phase wave. Schroedinger kept only the wave and, as with the interpretation that Heisenberg, Born, and Bohr provided for the matrix mechanics, regarded the notion of a particle trajectory as a superfluous concept.

By the time of the Solvay meeting, de Broglie had fully developed his *pilot wave* theory in a paper published in early 1927: "...*One then conceives the continuous wave as guiding the motion of the particle. It is a pilot wave*." De Broglie's theory included Schroedinger's equation (10) for the pilot wave. But to the Schroedinger equation, de Broglie added the guidance equation:

$$m\frac{d\boldsymbol{x}}{dt}=\nabla S \quad , \tag{11}$$

where *S* is the phase of the pilot wave:

$$\Psi = A\exp(iS/\hbar) \quad , \tag{12}$$

where *A* and *S* are real quantities.

The Schroedinger equation (10) describes the motion of the pilot wave, and the guidance equation (11) describes how the wave guides the particle. Note that (11) constitutes an overthrow of Newton's law of motion.

In the 1950s, Bohm recast de Broglie's theory into a second-order form designed to look like Newton's second law. Taking the time derivative of (11) and using (10) yields:

$$m\frac{d^2\boldsymbol{x}}{dt^2}=-\nabla(U+Q) \quad , \tag{13}$$

where

$$Q\equiv\frac{\hbar^2}{2m_0}\frac{\nabla^2 A}{A} \tag{14}$$

acts as a `quantum potential'. Equations (13) and (14), provided by Bohm, are mathematically equivalent to de Broglie's equations (10) and (11). Instead of the particle momentum being determined by the gradient of the pilot wave, Bohm has the particle acceleration being affected by the quantum potential. Thus the statement that de Broglie's theory is first order, and Bohm's theory is second order.

The mystery is why all this was lost to the mainstream history of science until now.

-L.L. Williams